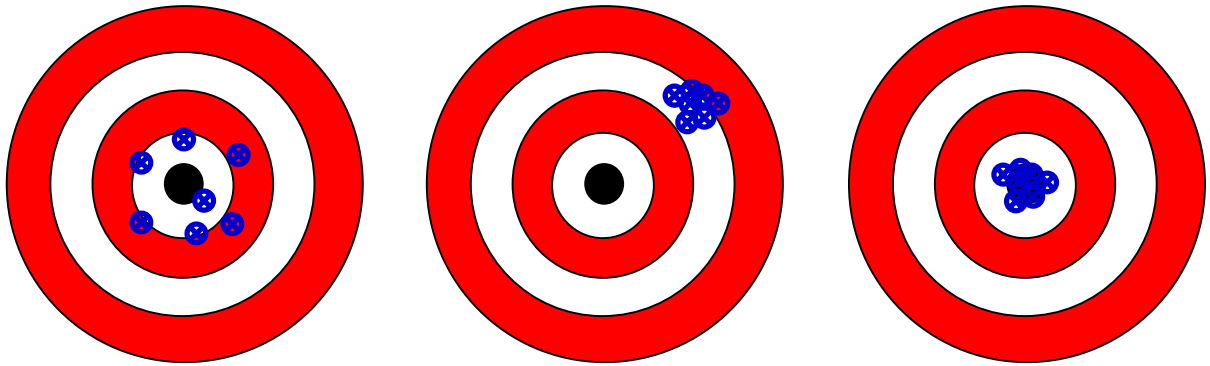
ADJUSTABLE GEOMETRIC CONSTRAINTS

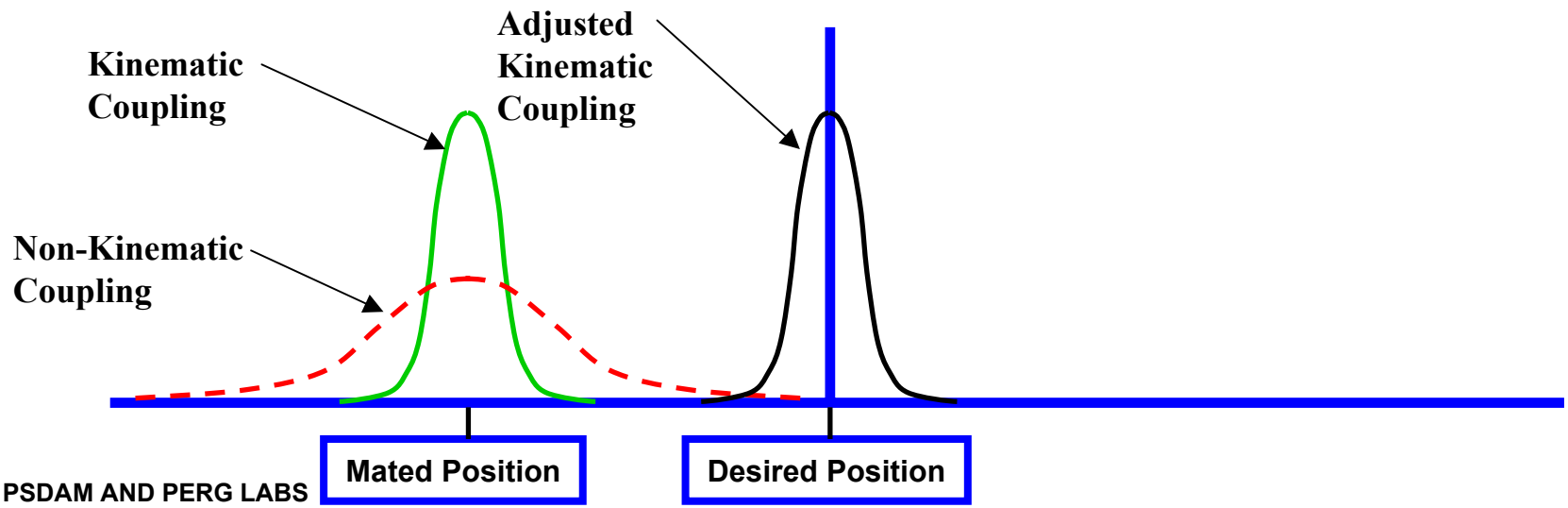
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Why adjust kinematic couplings?

KC Repeatability is orders of magnitude better than accuracy

Accuracy = *f* (manufacture and assemble)



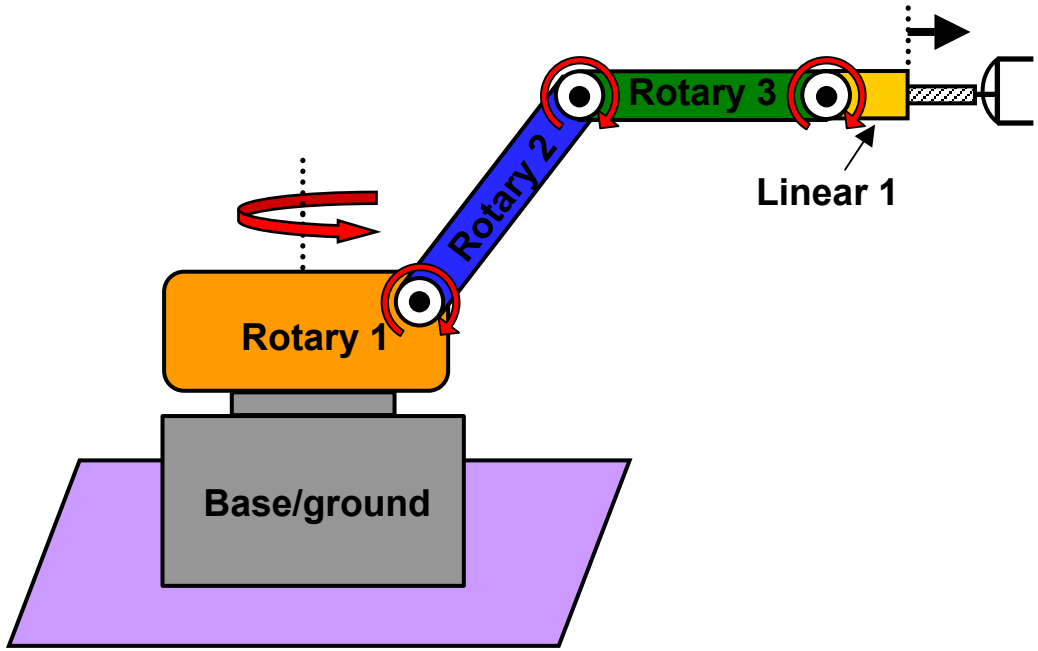


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Serial and parallel kinematic machines/mechanisms

SERIAL MECHANISMS

- Structure takes form of open loop
- I.e. Most mills, lathes, “stacked” axis robots
- Kinematics analysis typically easy



PARALLEL MECHANISMS

- Structure of closed loop chain(s)
- I.e. Stuart platforms & hexapods
- Kinematics analysis usually difficult
- 6 DOF mechanism/machine
- Multiple variations on this theme



Photo from Physik Instruments web page



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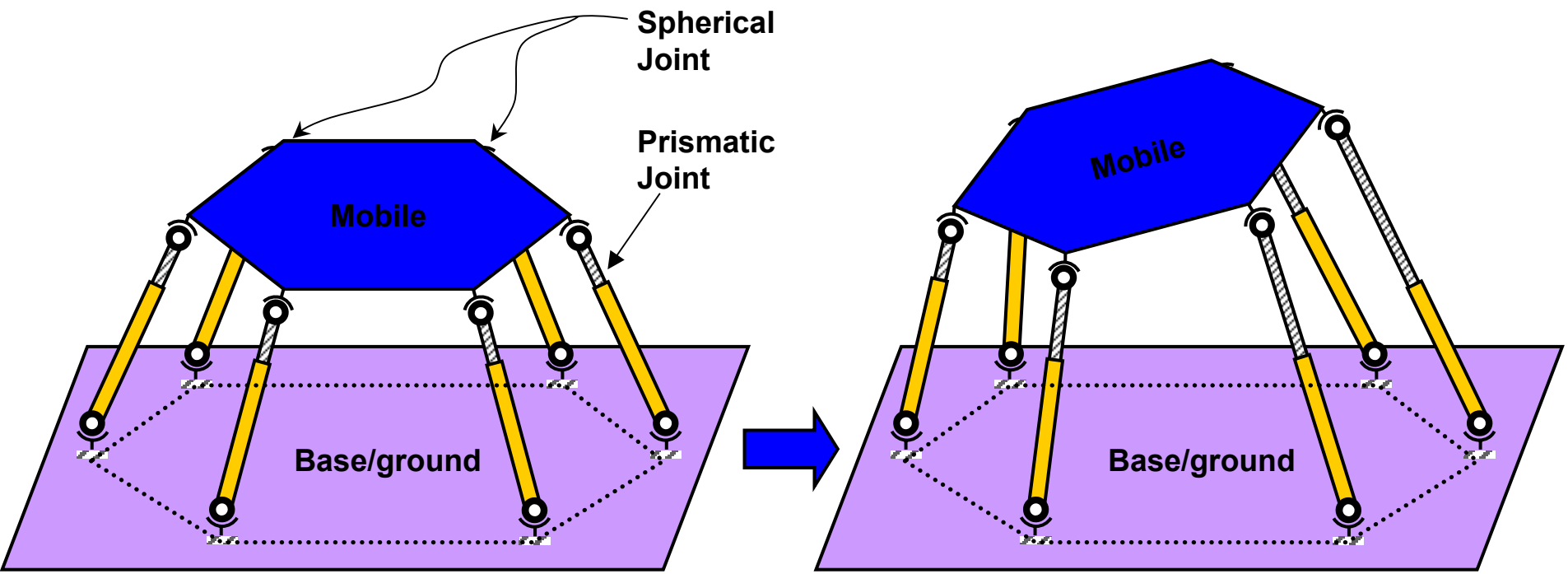
Parallel mechanism: Stewart-Gough platform

6 DOF mechanism/machine

Multiple variations on this theme with different joints:

⊙ Spherical joints:	3 Cs	Permits 3 rotary DOF	Ball Joint
⊙ Prismatic joints:	5 Cs	Permits one linear DOF	Sliding piston
⊙ Planar:	3 Cs	Permits two linear, one rotary DOF	Roller on plane

E.g. Changing length of "legs"

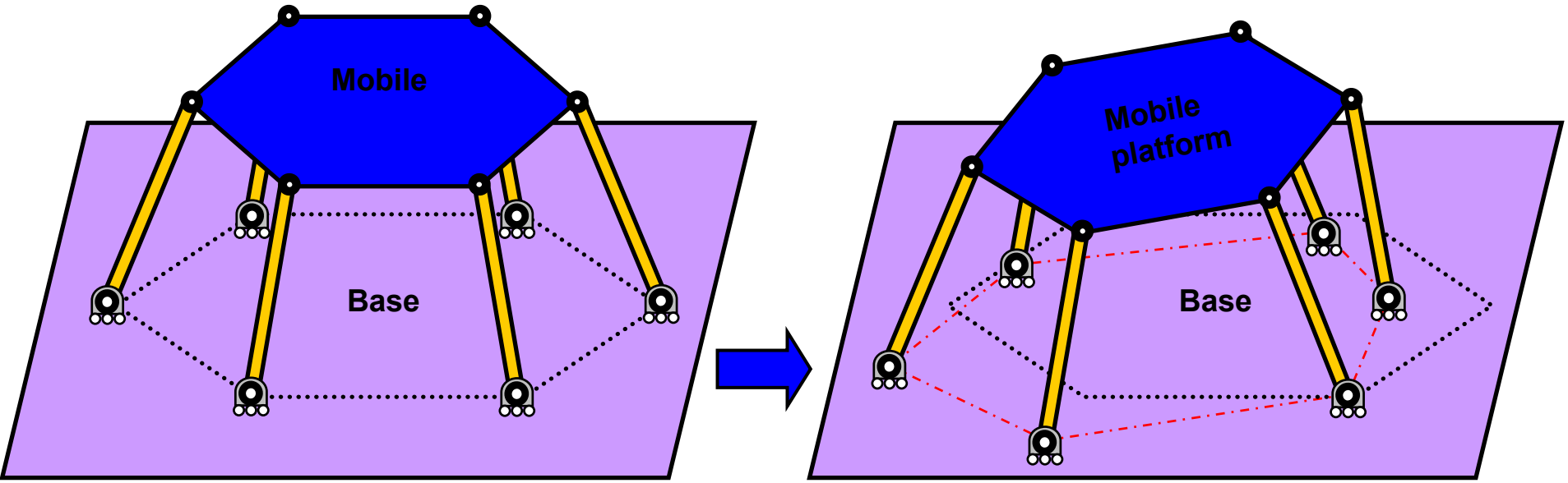


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Parallel mechanism: Variation

6 DOF mechanism/machine by changing position of joints

Can have a combination of position and length changes



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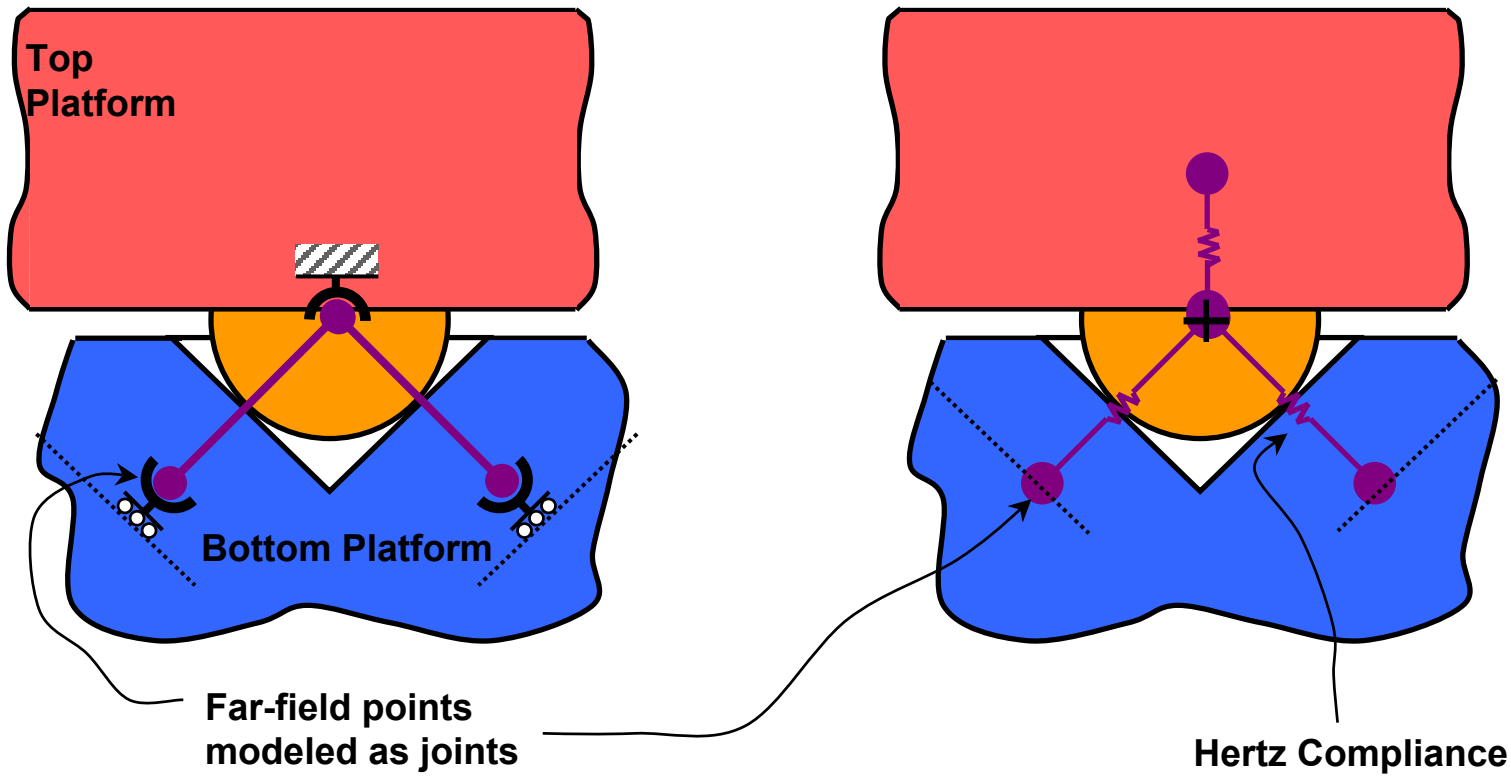
Kinematic couplings as mechanisms

Ideally, kinematic couplings are static parallel mechanisms

IRL, deflection(s) = mobile parallel kinematic mechanisms

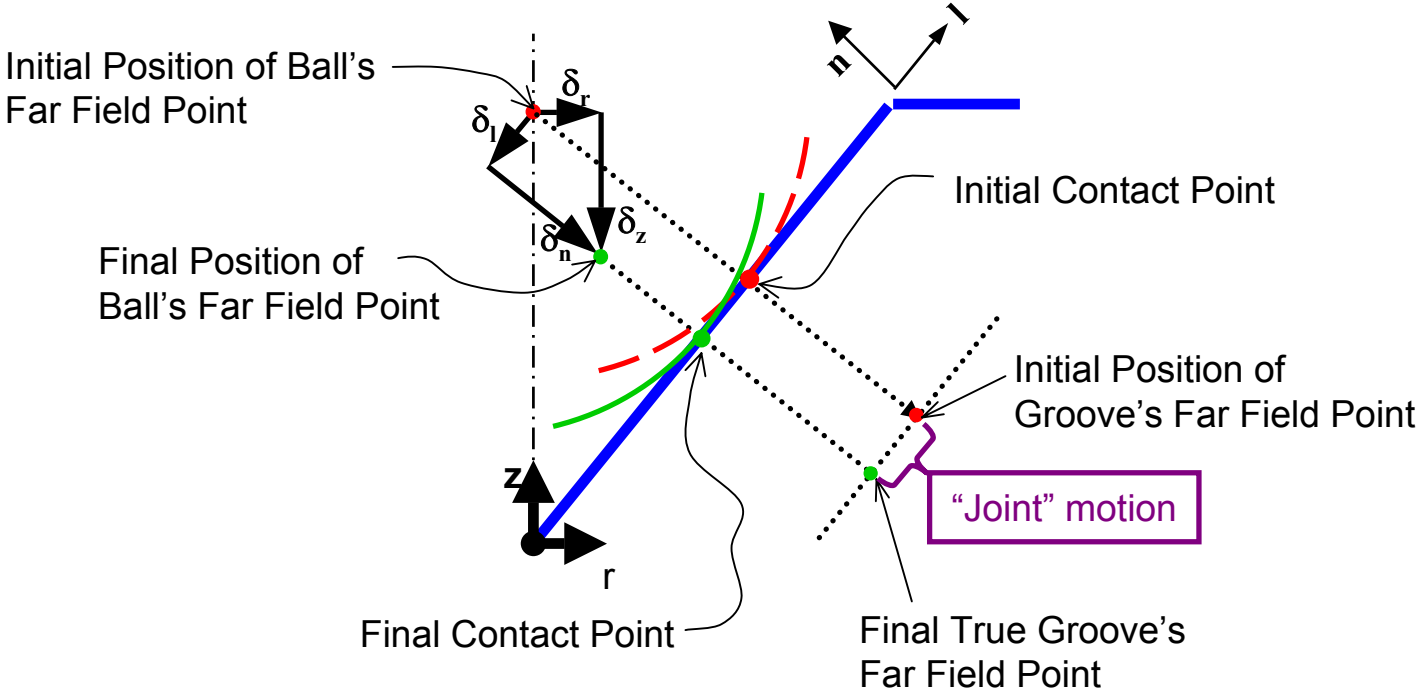
How are they “mobile”?

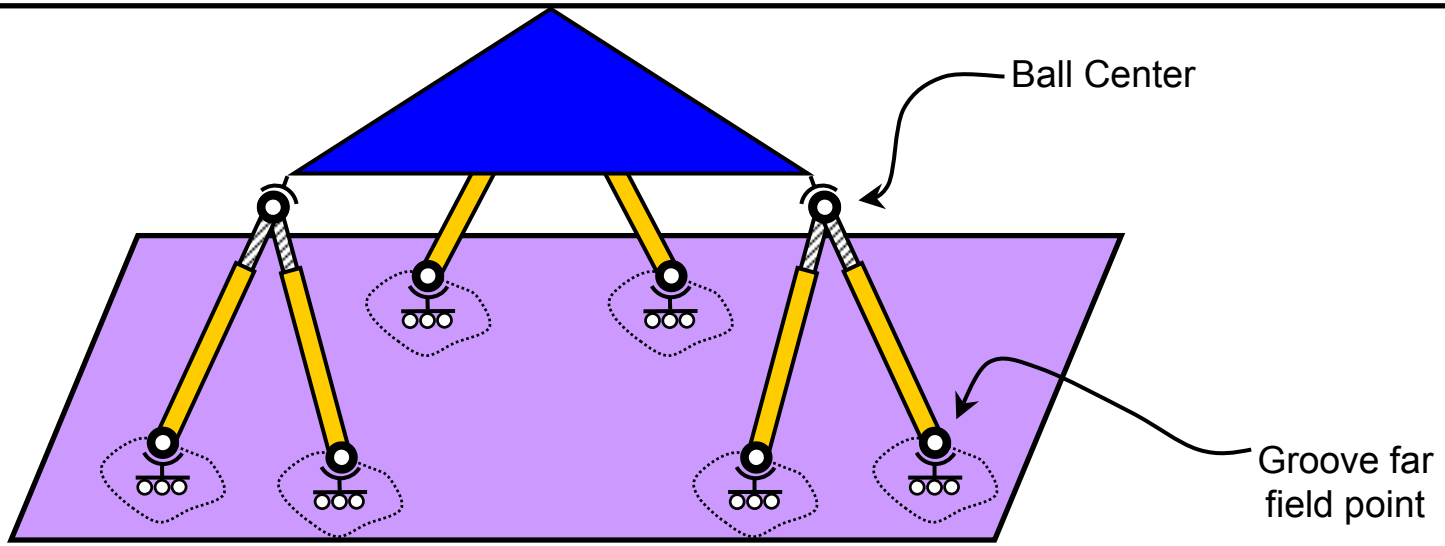
- Hertz normal distance of approach ~ length change of leg
- Far field points in bottom platform moves as ball center moves ~ joint motions



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Model of kinematic coupling mechanism





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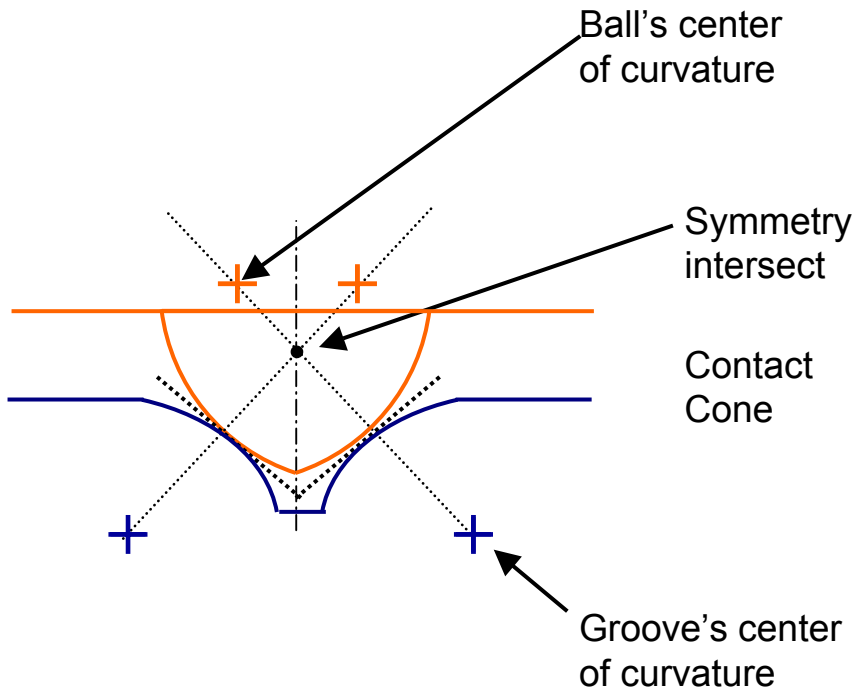
Accuracy of kinematic mechanisms

Since location of platform depends on length of legs and position of base and platform joints, accuracy is a function of mfg and assembly

Parameters affecting coupling centroid (platform) location:

- **Ball center of curvature location**
- **Ball orientation (i.e. canoe ball)**
- **Ball centerline intersect position (joint)**
- **Ball radii**

- **Groove center of curvature location**
- **Groove orientation**
- **Groove depth**
- **Groove radii**

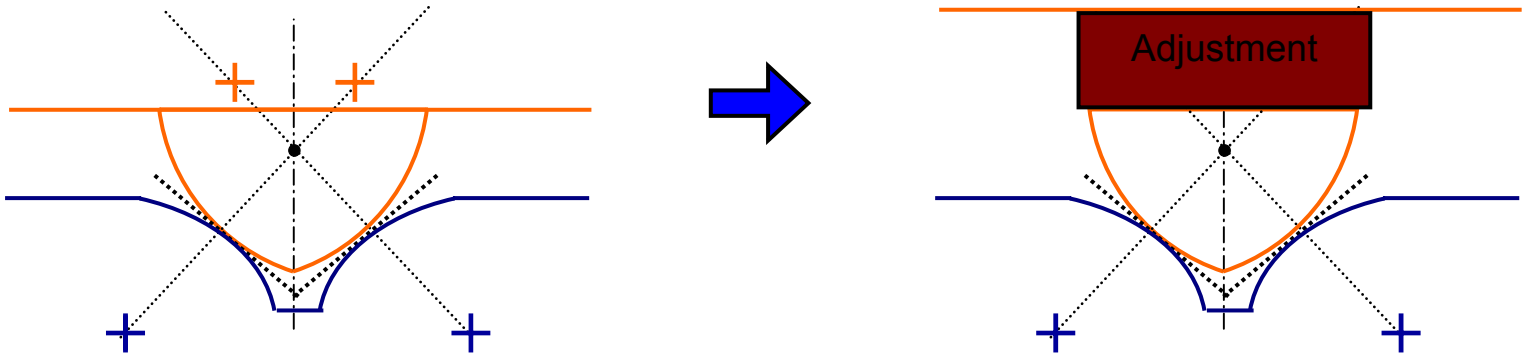


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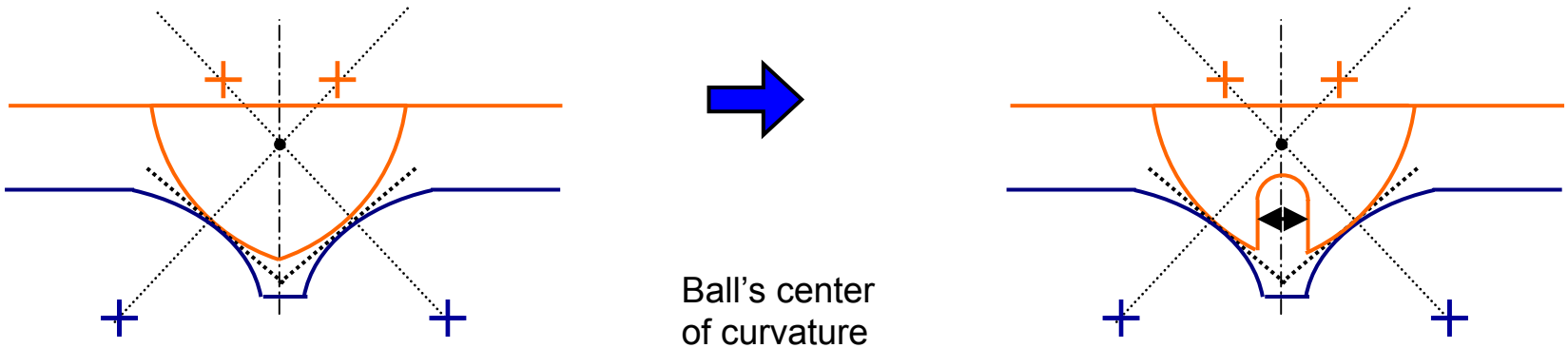
Utilizing the parallel nature of kinematic couplings

Add components that adjust or change link position/size, i.e.:

- Place adjustment between kinematic elements and platforms (joint position)



- Strain kinematic elements to correct inaccuracy (element size)

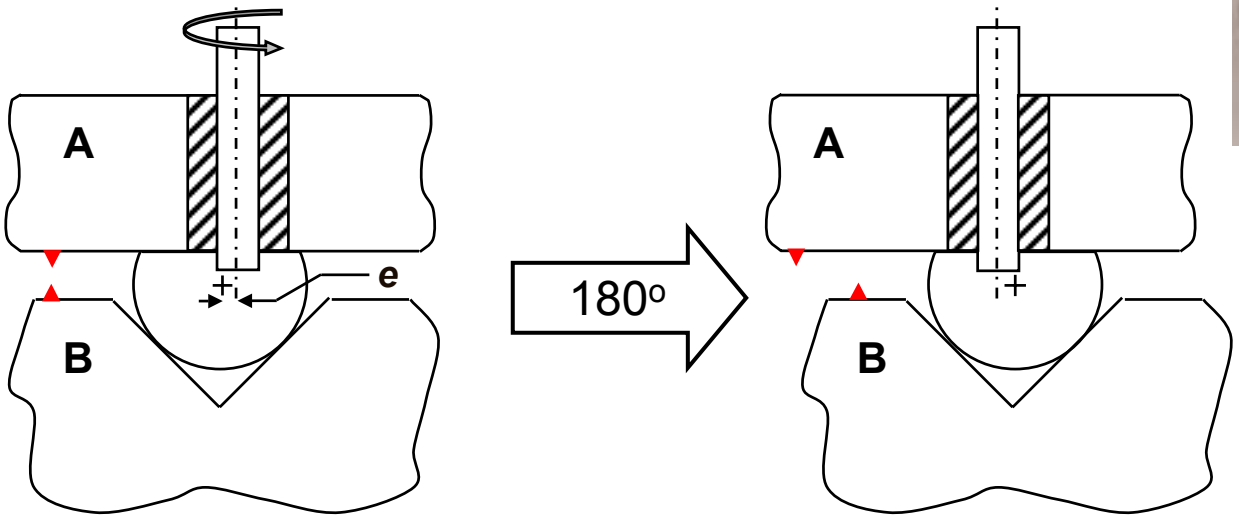


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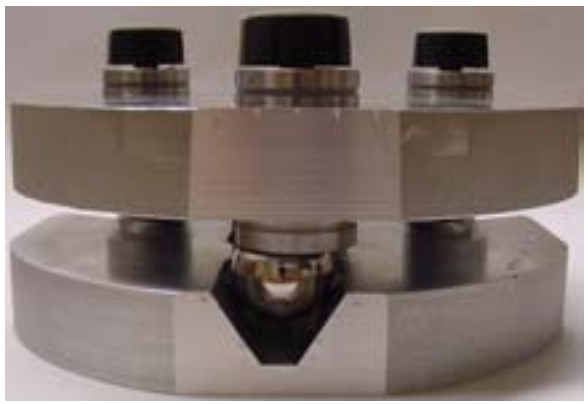
Example: Adjusting planar motion

Position control in x, y, θ_z:

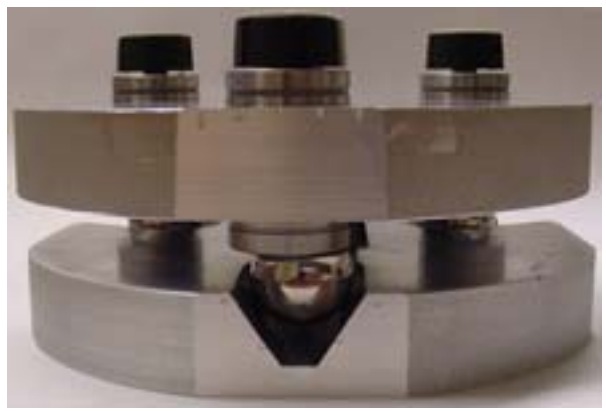
- Rotation axis offset from the center of the ball







Eccentric left



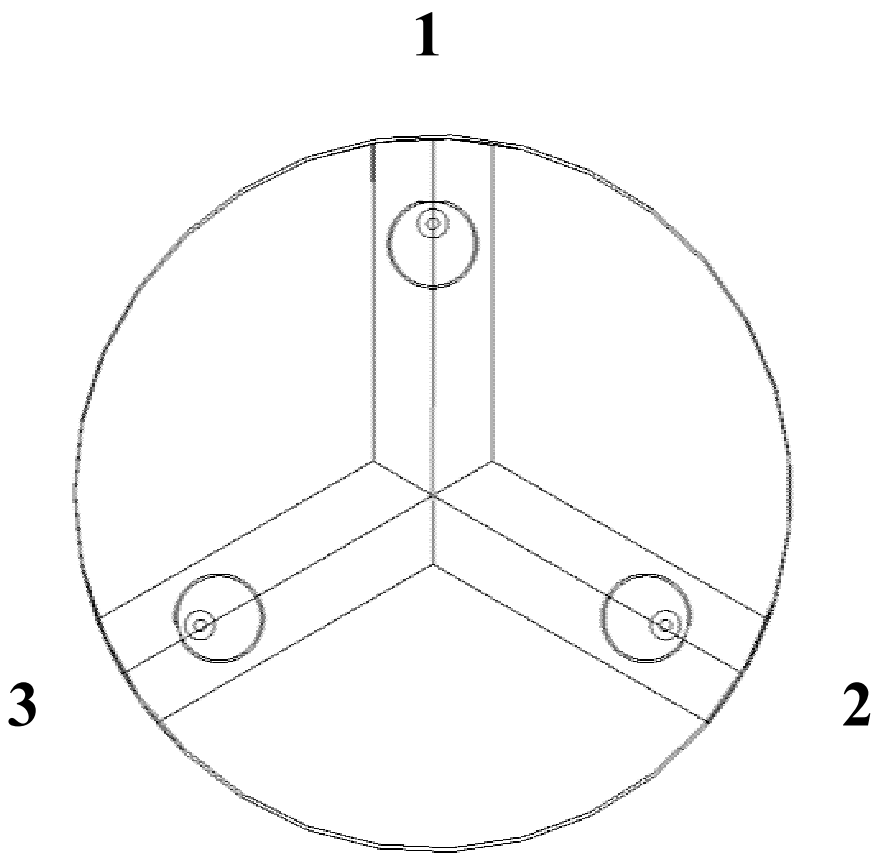
Eccentric right

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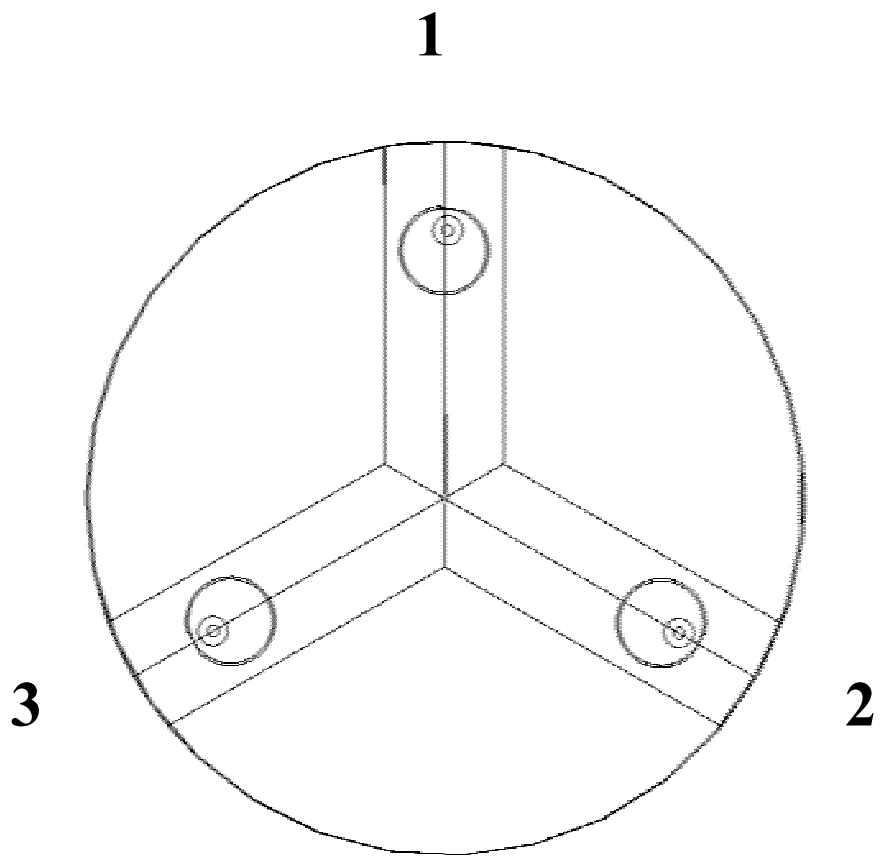
Patent Pending, Culpepper

ARKC demo animation





Input: Actuate Balls 2 & 3
Output: Δy



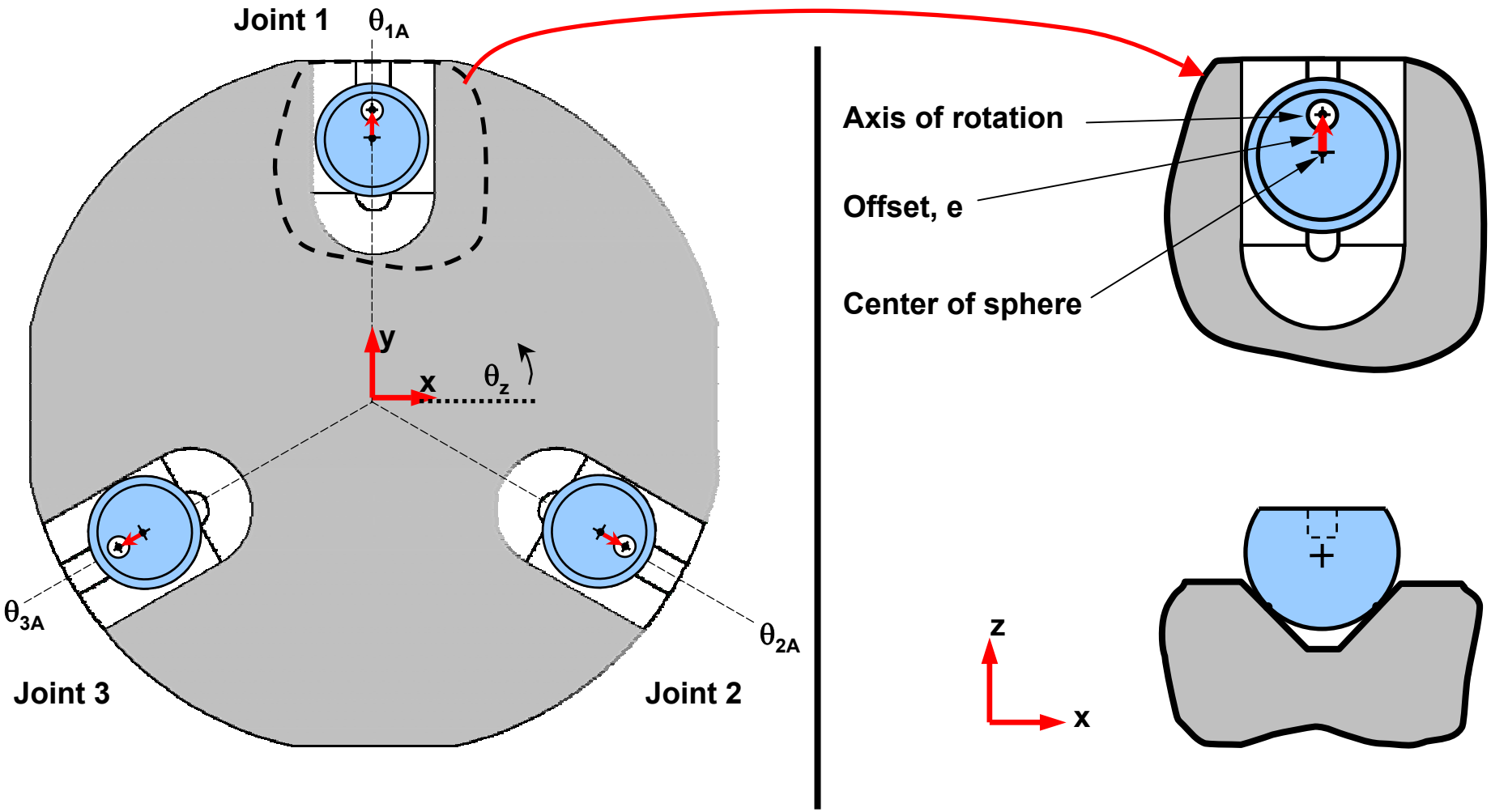
Input: Actuate Ball 1
Output: Δx and $\Delta\theta_z$

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Planar kinematic model

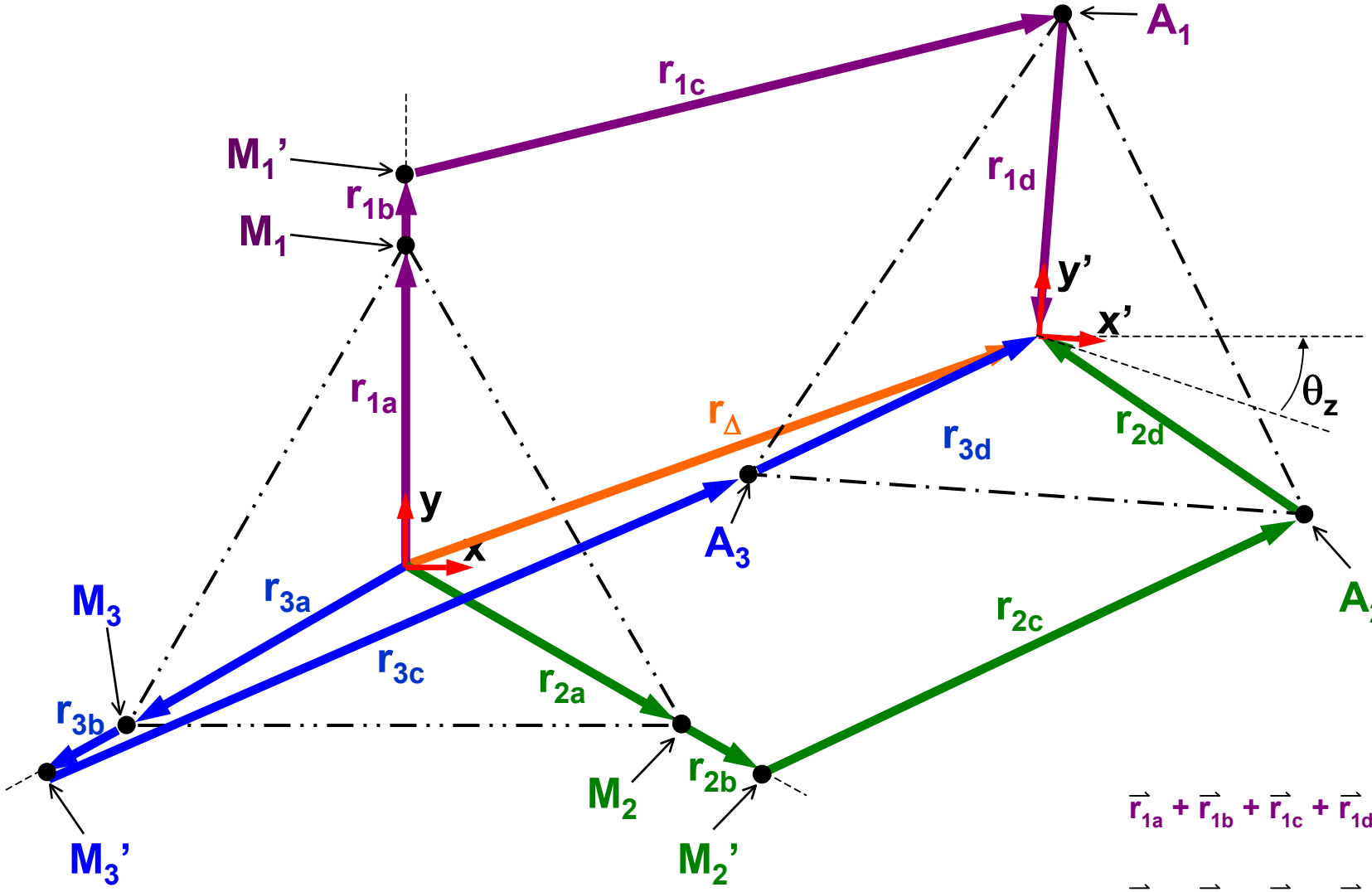
Equipping each joint provides control of 3 degrees of freedom

View of kinematic coupling with balls in grooves (top platform removed)



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Vector model for planar adjustment of KC



$$\vec{r}_{1a} + \vec{r}_{1b} + \vec{r}_{1c} + \vec{r}_{1d} = \vec{r}_{\Delta}$$

$$\vec{r}_{2a} + \vec{r}_{2b} + \vec{r}_{2c} + \vec{r}_{2d} = \vec{r}_{\Delta}$$

$$\vec{r}_{3a} + \vec{r}_{3b} + \vec{r}_{3c} + \vec{r}_{3d} = \vec{r}_{\Delta}$$

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Analytic position control equations

Vector equations:

$$\vec{r}_{1a} + \vec{r}_{1b} + \vec{r}_{1c} + \vec{r}_{1d} = \vec{r}_{\Delta}$$

$$\vec{r}_{2a} + \vec{r}_{2b} + \vec{r}_{2c} + \vec{r}_{2d} = \vec{r}_{\Delta}$$

$$\vec{r}_{3a} + \vec{r}_{3b} + \vec{r}_{3c} + \vec{r}_{3d} = \vec{r}_{\Delta}$$

System of 6 equations:

$$\begin{bmatrix} \cos\theta_{1A} & 0 & 0 & 1 & 0 \\ \sin\theta_{1A} & 0 & 0 & 0 & 1 \\ 0 & \cos\theta_{2A} & 0 & 1 & 0 \\ 0 & \sin\theta_{2A} & 0 & 0 & 1 \\ 0 & 0 & \cos\theta_{3A} & 1 & 0 \\ 0 & 0 & \sin\theta_{3A} & 0 & 1 \end{bmatrix} \begin{bmatrix} L_{1D}\sin\theta_{1A} \\ L_{1D}\cos\theta_{1A} \\ L_{2D}\sin\theta_{2A} \\ L_{2D}\cos\theta_{2A} \\ L_{3D}\sin\theta_{3A} \\ L_{3D}\cos\theta_{3A} \end{bmatrix} = \begin{bmatrix} L_{1B} \\ L_{2B} \\ L_{3B} \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} L_{1D}L_{1A}\cos\theta_{1A} + L_{1C}\cos\theta_{1C} \\ L_{1D}L_{1A}\sin\theta_{1A} + L_{1C}\sin\theta_{1C} \\ L_{2D}L_{2A}\cos\theta_{2A} + L_{2C}\cos\theta_{2C} \\ L_{2D}L_{2A}\sin\theta_{2A} + L_{2C}\sin\theta_{2C} \\ L_{3D}L_{3A}\cos\theta_{3A} + L_{3C}\cos\theta_{3C} \\ L_{3D}L_{3A}\sin\theta_{3A} + L_{3C}\sin\theta_{3C} \end{bmatrix}$$

For standard coupling: 120°; offset E; coupling radius R; $\sin(\theta_z) \sim \theta_z$:

$$x \cong \frac{E}{3}\left(-2\sin(\theta_{1C}-\theta_{1A}) + \sin(\theta_{2C}-\theta_{2A}) + (\theta_{3C}-\theta_{3A})\right)$$

$$y \cong \frac{\sqrt{3}E}{3}\left(\sin(\theta_{2C}-\theta_{2A}) - (\theta_{3C}-\theta_{3A})\right)$$

$$z \cong \frac{1}{3}\left(z_1 + z_2 + z_3\right)$$

$$\theta_X \cong -\frac{1}{3}\left(2z_1 - z_2 - z_3\right)$$

$$\theta_Y \cong -\frac{\sqrt{3}}{3}\left(z_2 - z_1\right)$$

$$\theta_Z \cong \frac{-1}{3}\frac{E}{R}\left(\sin(\theta_{1C}-\theta_{1A}) + \sin(\theta_{2C}-\theta_{2A}) + (\theta_{3C}-\theta_{3A})\right)$$

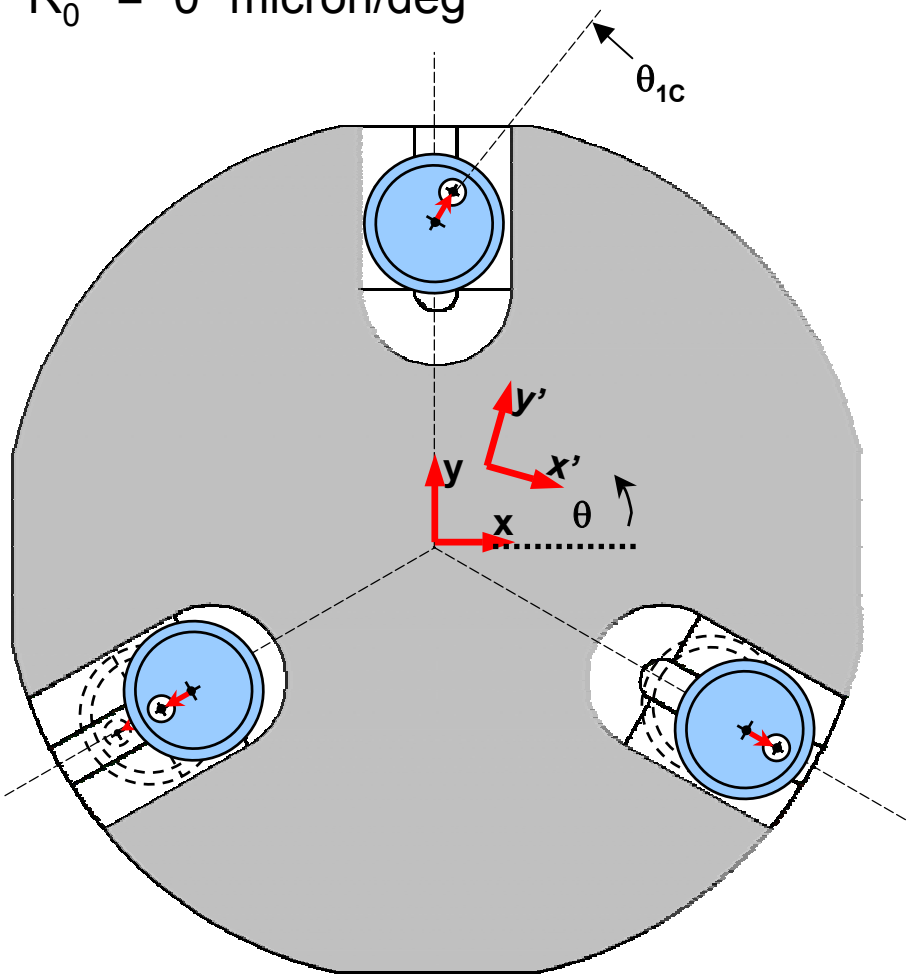
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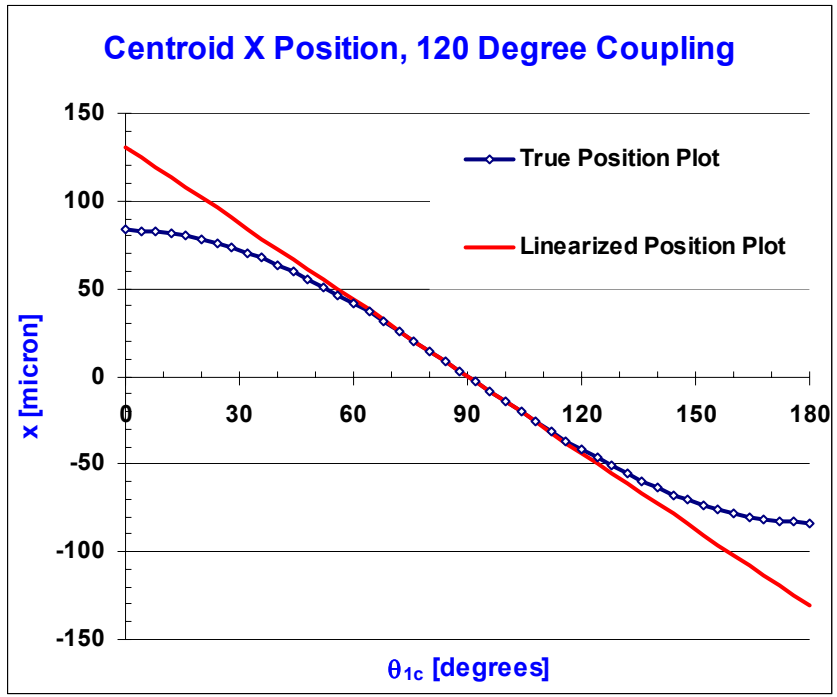
ARKC resolution analysis

For E = 125 microns

R_{90} = -1.5 micron/deg

R_0 = “0” micron/deg





Limits on Linear Resolution Assumptions

% Error	θ_{1C} Lower Limit [Degree]	θ_{1C} Upper Limit [Degree]	θ_{1C} Half Range [Degree]
1	75	105	+/- 15
2	70	110	+/- 20
5	60	120	+/- 30
10	47	133	+/- 43

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Forward and reverse kinematic solutions

PART I: COUPLING CHARACTERISTICS

Use this to specify groove angles and input ball rotations and heights used to calculate coupling position in part II

θ_{1A}	90	degrees	z_1	5	microns
	1.571	radians		0.0001969	inches
θ_{2A}	330	degrees			
	5.760	radians	z_2	500	microns
$\theta 3_A$	210	degrees		0.0196850	inches
	3.665	radians			
			z_3	200	microns
θ_{1C}	87.7073	degrees		0.0078740	inches
	1.531	radians			
θ_{2C}	331.1455	degrees			
	5.780	radians			
$\theta 3_C$	211.1467	degrees			
	3.685	radians			
E	125	microns			
	0.0049	inches			
R_T	57150	microns	Note: $R_T = L_{iD}$		
	2.2500	inches			

PART II: CALCULATED MOVEMENTS

This takes input from part I to calculate the position of the top part of the coupling.

θ_Z	0.0000	radians	θ_X	-0.004024	radians
	0.0006	μradians		-4024.4889	μradians
	0.000000	degrees		-0.230586	degrees
x	5.0005	microns	θ_Y	-0.003031	radians
	0.000197	inches		-3030.7040	μradians
				-0.173647	degrees
y	-0.0014	microns	z	235.0000	microns
	0.000000	inches		0.009252	inches

PART III: REVERSE SOLUTION

Use this to input a desired position and goal seek to solve for the position of the balls

	DESIRED POSITION	POSITION ERROR		
θ_Z	0.000	0.00	0.0	μrad
x	5.000	-0.001		microns
y	0.000	0.001		microns

BALL SETTINGS

See part I for modified ball angles, the following angles are the difference between groove and ball angles

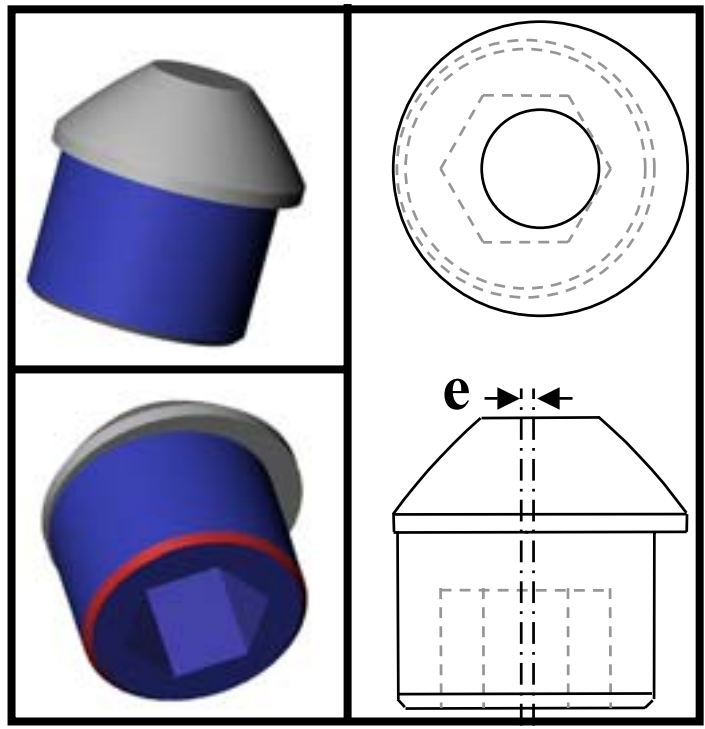
θ_1	-2.2927	degrees
θ_2	1.1455	degrees
$\theta 3$	1.1467	degrees

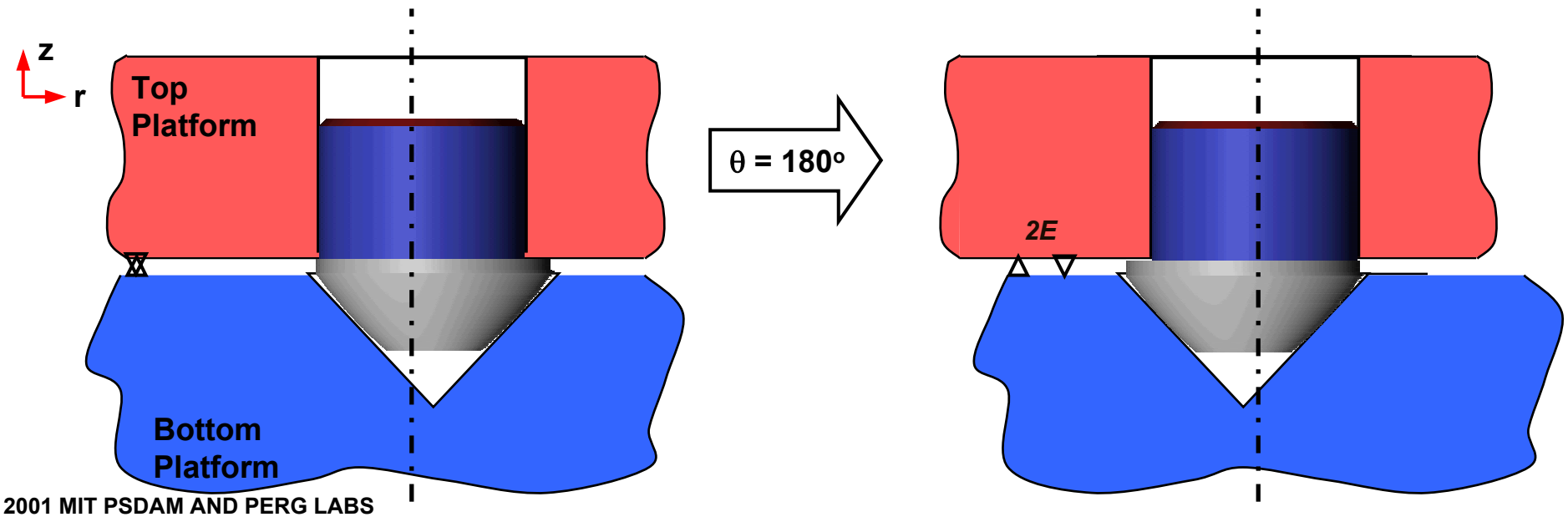
ERROR SUM 2698.1 <- x, y, θ_z <- Use the solver on this value to set it to a value (ideally zero) that is small, but greater than or equal to 0.

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Low-cost adjustment (10 μm)

- Peg shank and convex crown are offset
- Light press between peg and bore in plate
- Adjustment with allen wrench
- Epoxy or spreading to set in place
- Friction (of press fit) must be minimized…





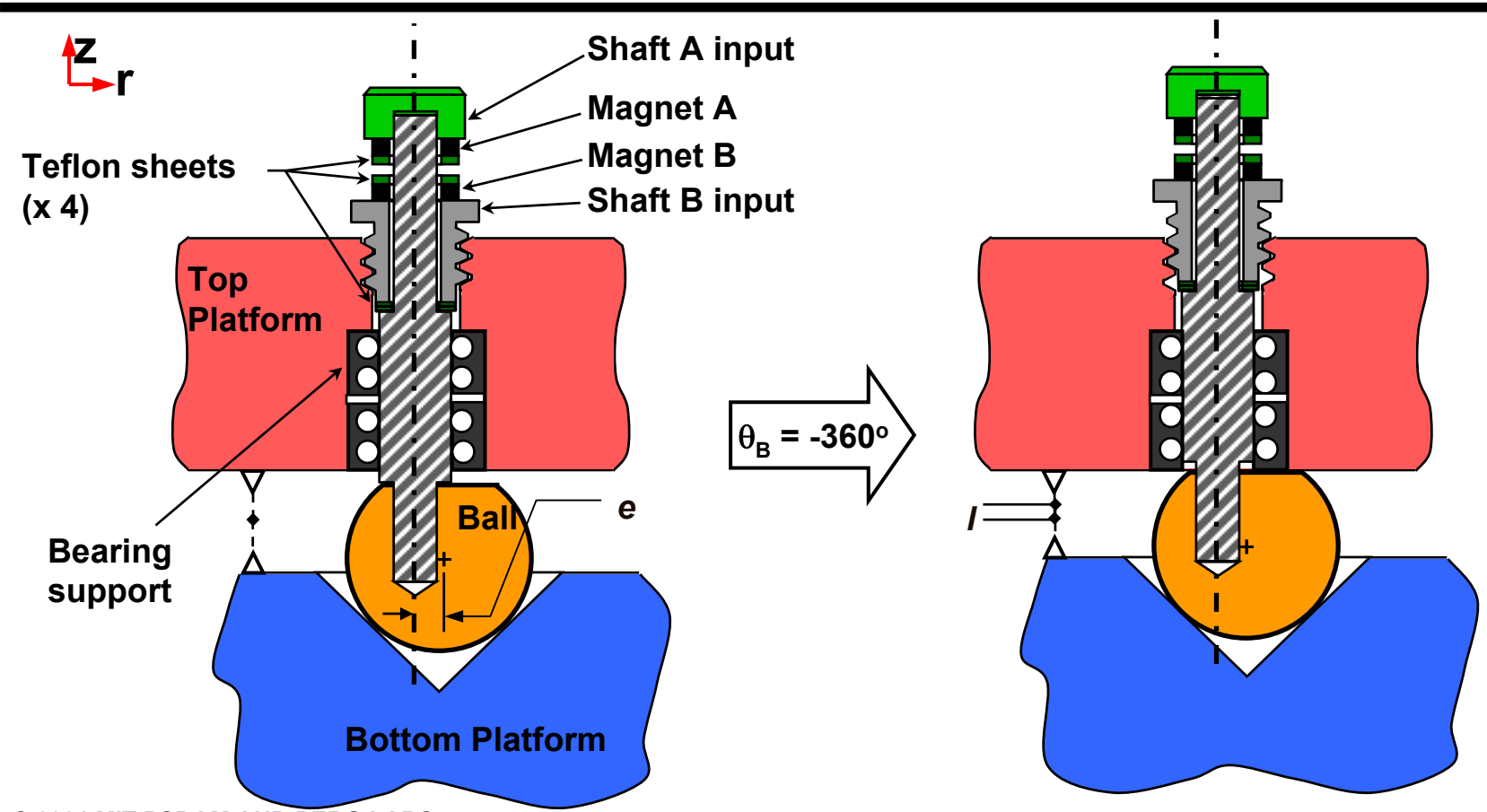
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Moderate-cost adjustment (3 micron)

Shaft B positions z height of shaft A [z, θ_x, θ_y]

Shaft A positions as before [x, y, θ_z]

Force source preload I.e. magnets, cams, etc..



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“Premium” adjustment (sub-micron)

z
r

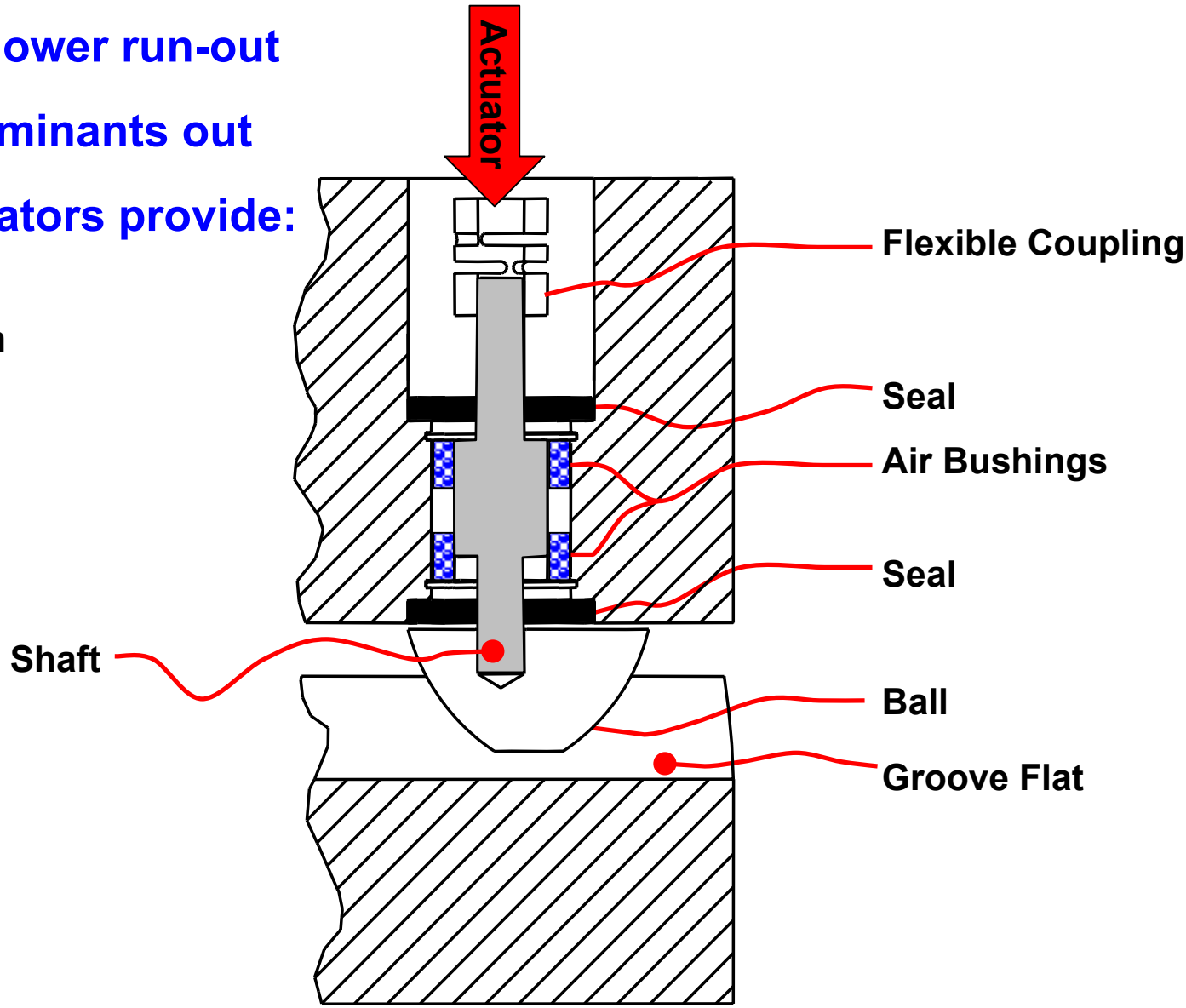
Run-out is a major cause of error

Air bushings for lower run-out

Seals keep contaminants out

Dual motion actuators provide:

- Linear motion
- Rotary motion

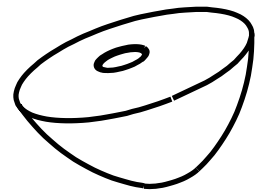


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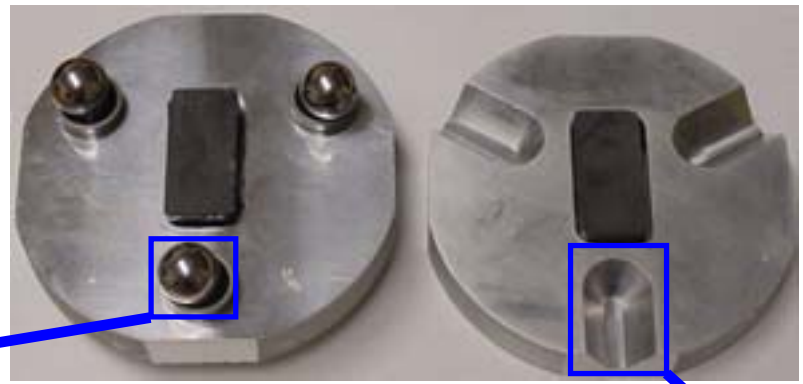
Mechanical interface wear management

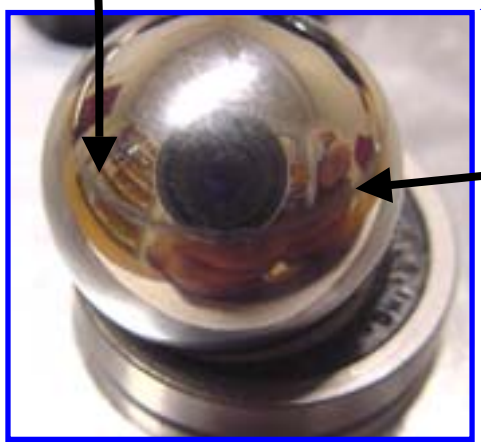
Wear and particle generation are unknowns. Must investigate:

- Coatings [minimize friction, maximize surface energy]
- Surface geometry, minimize contact forces
- Alternate means of force/constraint generation

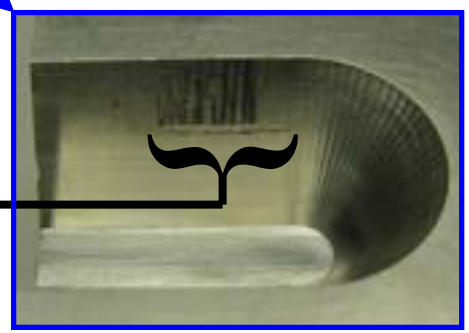


At present, must uncouple before actuation





Sliding damage



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