

Topic 3
*FUN*da*MENTAL* Principles

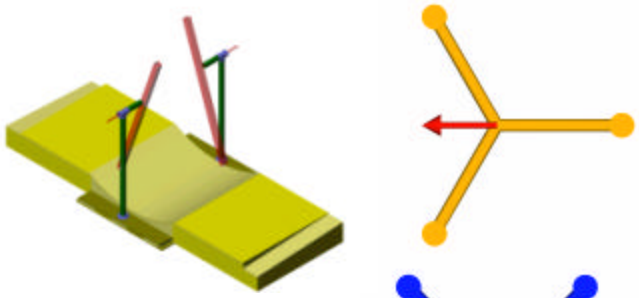


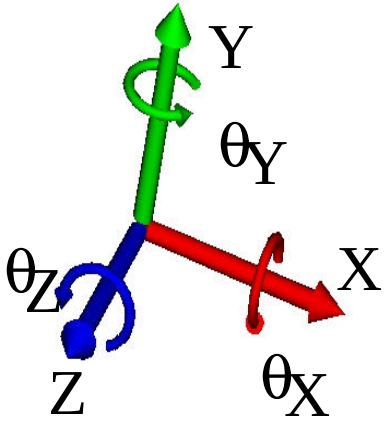


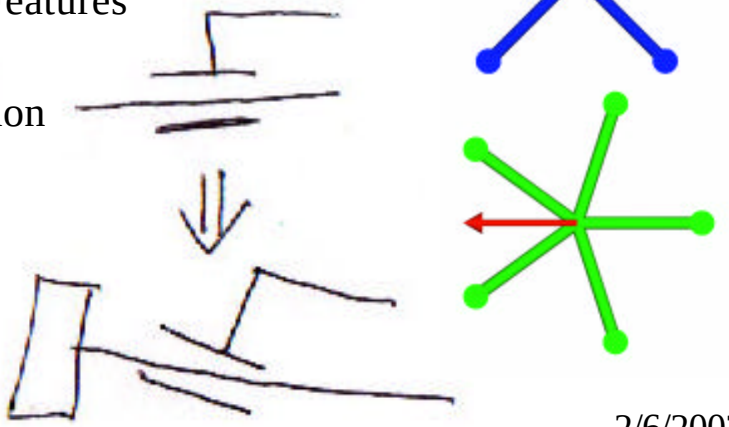
Topics

- Occam's Razor
- Saint-Venant's Principle
- Golden Rectangle
- Abbe's Principle
- Maxwell & Reciprocity
- Self-Principles
- Stability
- Symmetry
- Parallel Axis Theorem
- Accuracy, Repeatability, Resolution
- Sensitive Directions & Reference Features
- Structural Loops
- Free Body Diagrams & Superposition
- Centers of Action
- Exact Constraint Design
- Elastically Averaged Design
- Stick Figures









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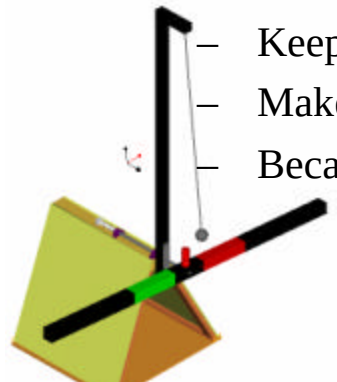
Occam's Razor

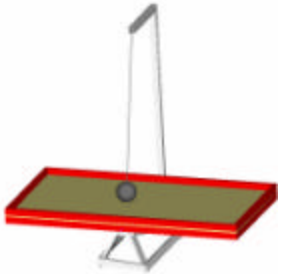
- William of Occam (or Ockham) (1284-1347) was an English philosopher and theologian
  - Ockham stressed the Aristotelian principle that *entities must not be multiplied beyond what is necessary*
  - "Ockham wrote fervently against the Papacy in a series of treatises on Papal power and civil sovereignty. The medieval rule of parsimony, or principle of economy, frequently used by Ockham came to be known as **Ockham's razor**. The rule, which said that *plurality should not be assumed without necessity* (or, in modern English, *keep it simple, stupid*), was used to eliminate many pseudo-explanatory entities"
    (http://wotug.ukc.ac.uk/parallel/www/occam/occam-bio.html)

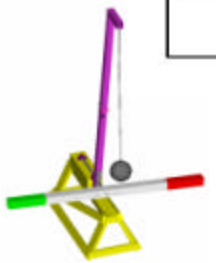


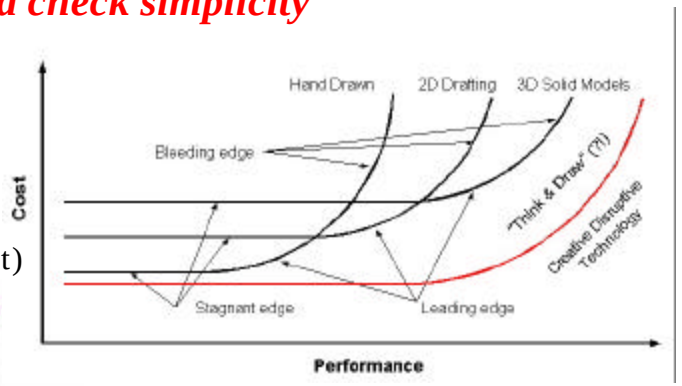
- ***A problem should be stated in its most basic and simplest terms***
- ***The simplest theory that fits the facts of a problem is the one that should be selected***
- ***Limit Analysis is an invaluable way to identify and check simplicity***

- Use fundamental principles as catalysts to help you
  - Keep It Super Simple (KISS)
  - Make It Super Simple (MISS)
  - Because "*Silicon is cheaper than cast iron*" (Don Blomquist)



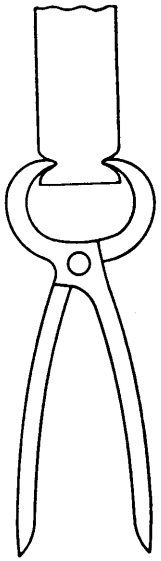


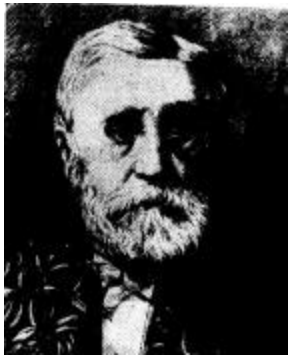




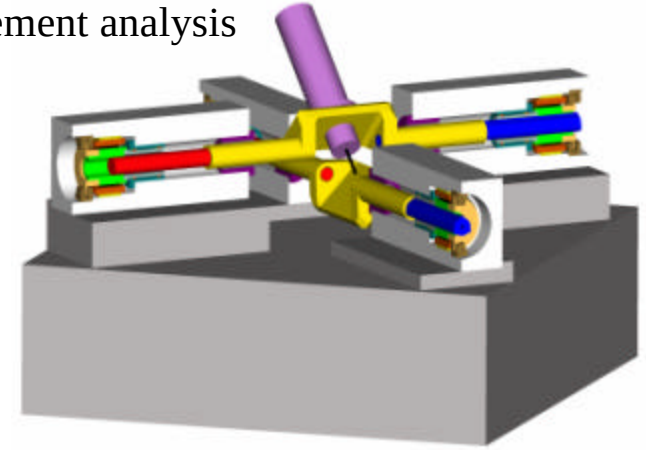
Saint-Venant's Principle

- Saint-Venant's Principle
  - Saint-Venant did extensive research in the theory of elasticity, and many times he relied on the assumption that local effects of loading do not affect global strains
    - e.g., bending strains at the root of a cantilever are not influenced by the local deformations of a point load applied to the end of a cantilever
  - The engineering application of his general observations are profound for the development of conceptual ideas and initial layouts of designs:
    - To NOT be affected by local deformations of a force, be several characteristic dimensions away
      - How many seats away from the sweaty dude do you want to be?
      - Several can be interpreted as 3-5
    - To have control of an object, apply constraints over several characteristic dimensions
      - These are just initial layout guidelines, and designs must be optimized using closed-form or finite element analysis





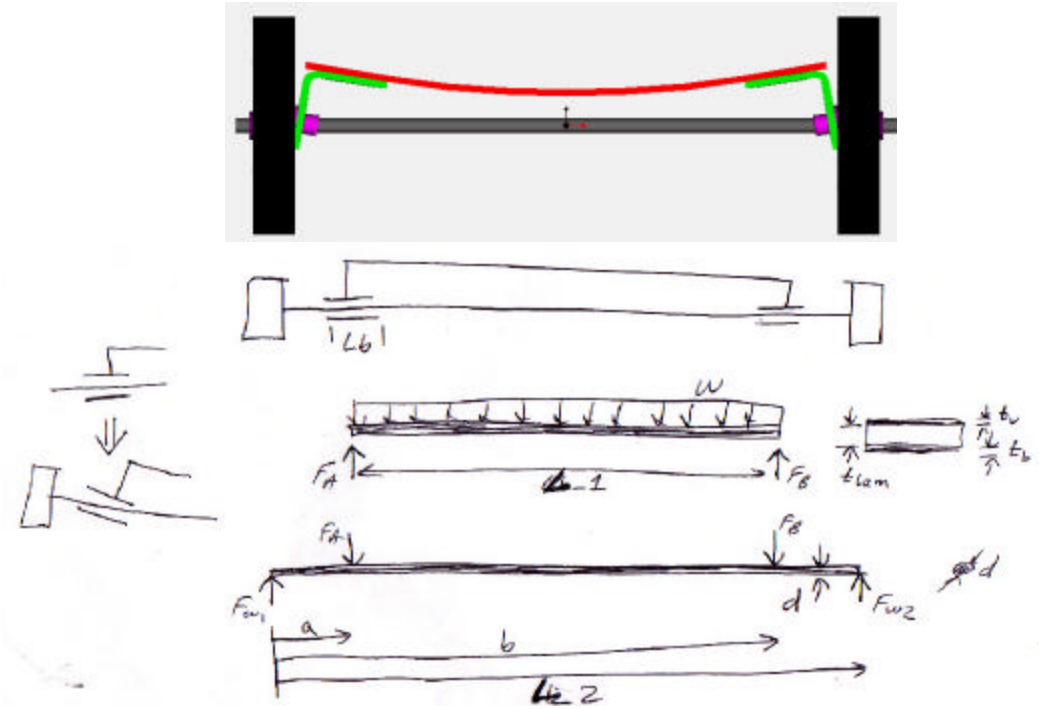
Barré de Saint-Venant 1797-1886



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Saint-Venant's Principle: *Structures*

- To NOT feel something's effects, be several characteristic dimensions away!
  - If a plate is 5 mm thick and a bolt passes through it, you should be 3 plate thicknesses away from the bolt force to not cause any warping of the plate!
    - Many bearing systems fail because bolts are too close to the bearings
- To DOMINATE and CONTROL something, control several characteristic dimensions
  - If a column is to be cantilevered, the anchor region should be 3 times the column base area
  - Most machines that suffer from "lawn furniture syndrome" have inadequate proportions
    - Diagonal braces or gussets, that are 3-5 x the column base width, can make a column appear to be cantilevered

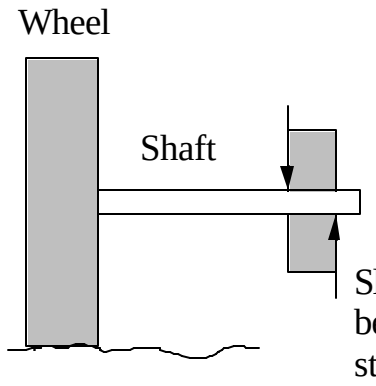


Axledefl.xls: Design Parameters	thin plate	laminate
Number of support axles, N	**2**	**2**
Bearing length, Lb (m)	**0.01**	**0.01**
Bottom beam length, L_2 (m)	**0.25**	**0.25**
Axle diameter, d (m)	**0.006**	**0.006**
Axle modulus, Eaxle (Pa)	**2.00E+11**	**2E+11**
Total load on top beam, F (N)	**50**	**50**
Top beam length, L_1 (m)	**0.2**	**0.2**
Distance wheels to 1st bearing, a (m)	**0.025**	**0.025**
Distance wheels to 2nd bearing, b (m)	**0.225**	**0.225**
Top beam top layer thickness, tu (m)	**0.0015**	**0.0015**
Top beam bottom layer thickness, tb (m)	**0**	**0.0015**
Top beam laminate spacer thickness, tlam (m)	**0**	**0.01**
Top beam front-to-back width, width (m)	**0.3**	**0.3**
Top beam layer modulus, E (Pa)	**7.00E+10**	**7.00E+10**
Top beam EI per axle, EI (N-m^2)	**3**	**1047**
Axle EI, EIaxle (N-m^2)	**13**	**13**
Load per unit width, w (N/m)	**250**	**250**
Upper beam slope at bearings, alpha1 (rad)	**-0.0282**	**-0.0001**
Axle slope at bearings, alpha2 (rad)	**-0.0049**	**-0.0049**
Net slope at bearings, alphabear (rad)	**0.0233**	**-0.0048**
Change in bearing diametral clearance, delta (mm)	**0.2331**	**-0.0483**

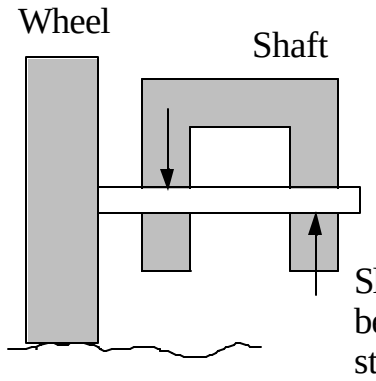
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Saint-Venant's Principle: *Bearings*

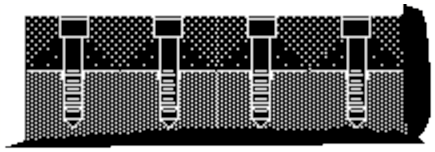
- Saint-Venant: *Linear Bearings:*
  - Make friction (μ) low and L/D>1, 1.6:1 very good, 3:1 awesome
  - Every year some students try L/D<1 and their machines jam!
    - Wide drawers guided at the outside edges can jam
    - Wide drawers guided by a central runner do not!
    - If L/D<1, actuate both sides of the slide!
- Saint-Venant: *Rotary Bearings:*
  - L/D>3 if you are to have the bearings "build the shaft into a wall"
  - IF L/D<3, BE careful that slope from shaft bending does not KILL the bearing!
- Bolting bearings in pace: beware the zone under a bolt that deforms due to bolt pressure!

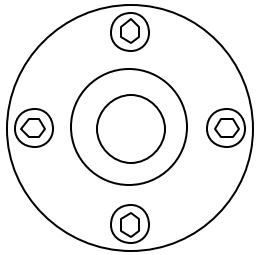


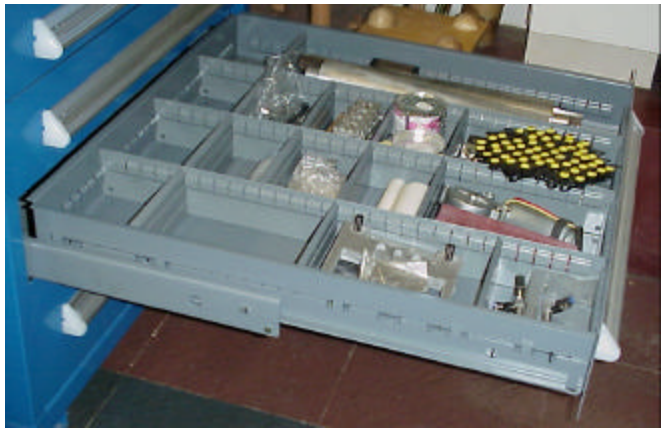
!!Non Optimal!!

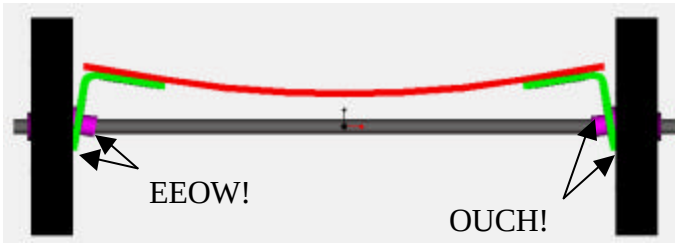


!!Optimal!!





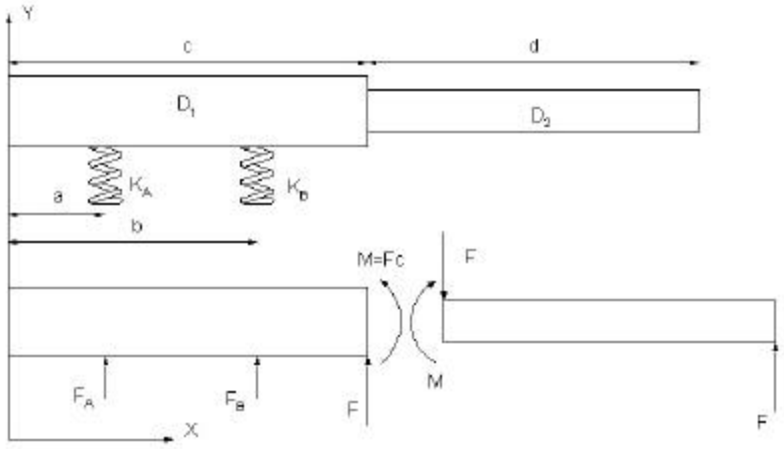




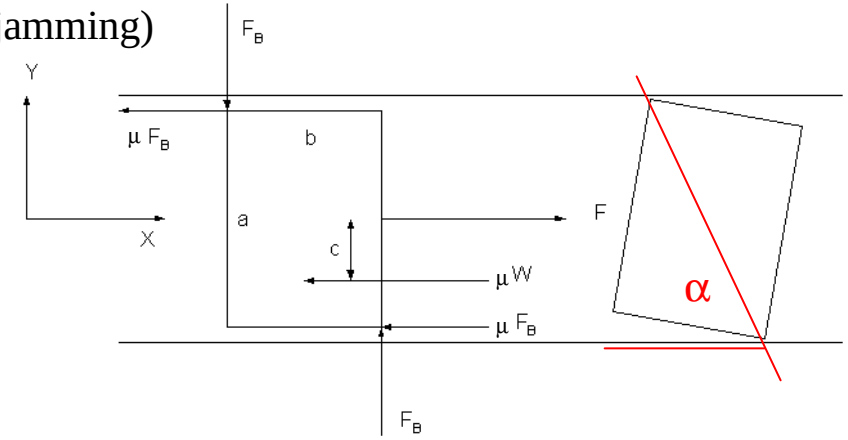
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Saint-Venant's Principle: *Bearings*

- Model of a shaft supported by bearings where we want to minimize the deflection of one end subject to error motions (unwanted radial forces) at the other end:



- Model of the effect of bearing width, friction, and length spacing on the actuation force (drawer jamming)

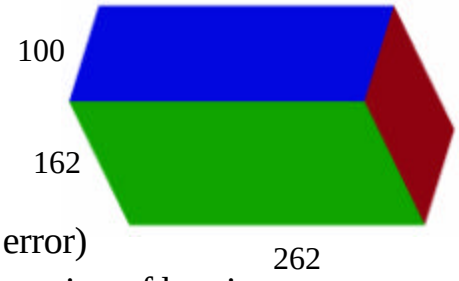


D1D2beam.xls: Inputs	
a	**0.05**
b	**0.2**
cc	**0.25**
d	**0.1**
D_1	**0.05**
D_2	**0.025**
Spring, KA	**2.00E+08**
Spring, KB	**2.00E+08**
Modulus, E	**2.00E+11**
Tip force, F	**1.00**
Outputs	
Moment of inertia, I_1	**3.07E-07**
Moment of inertia, I_2	**1.92E-08**
Spring force, FA	**1.00**
Spring force, FB	**-2.00**
Constant of integation, C_1	**2.39E-03**
Constant of integation, C_2	**-4.26E-04**
End deflection of just D_2 segment	**8.69E-08**
End slope of just D_2 segment	**1.30E-06**
Ratio $\delta(0)/\delta(c+d)$	**-0.049**

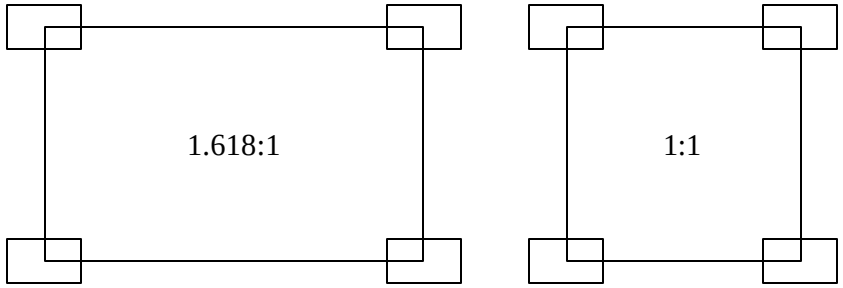
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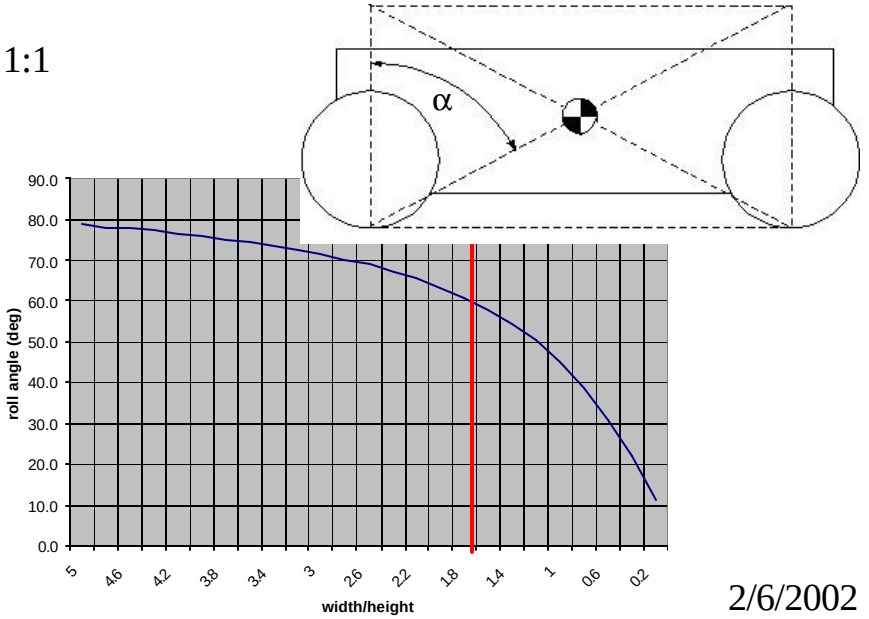
The Golden Rectangle

- The proportions of the *Golden Rectangle* are a natural starting point for preliminary sizing of structures and elements
  - *Golden Rectangle:* A rectangle where when a square is cut from the rectangle, the remaining rectangle has the same proportions as the original rectangle: a/1 = 1/(a-1)
    - See and study *Donald in Mathmagic Land!*
  - Try a *Golden Solid:* 1: 1.618: 2.618, & the diagonal has length 2a = 3.236
  - Example: Bearings:
  - The greater the ratio of the longitudinal to latitudinal (length to width) spacing:
    - The smoother the motion will be and the less the chance of walking (yaw error)
    - First try to design the system so the ratio of the longitudinal to latitudinal spacing of bearing elements is about 2:1
    - For the space conscious, the bearing elements can lie on the perimeter of a golden rectangle (ratio about 1.618:1)
    - The minimum length to width ratio should be 1:1
    - To minimize yaw error
    - Depends on friction too









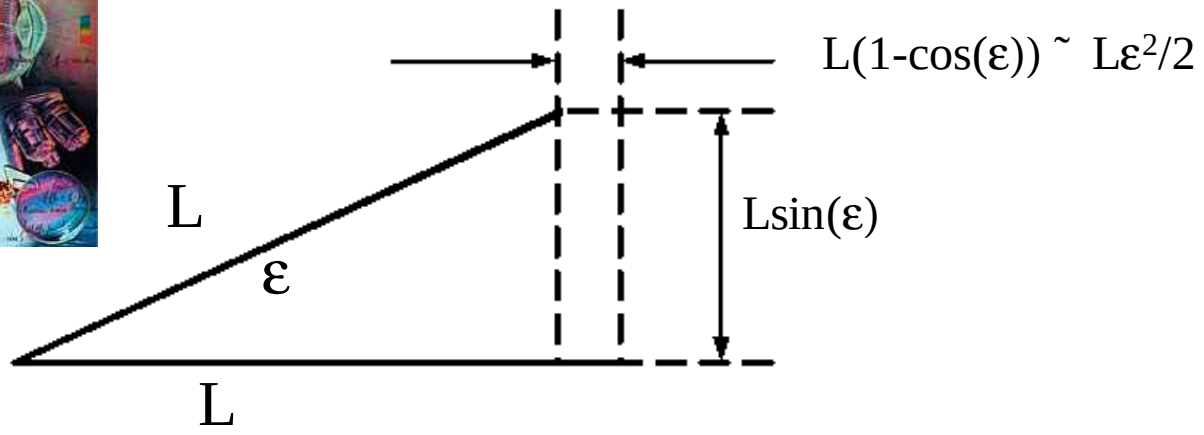
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Abbe's Principle

- In the late 1800s, Dr. Ernst Abbe (1840-1905) and Dr. Carl Zeiss (1816-1888) worked together to create one of the world's foremost precision optics companies: Carl Zeiss, GmbH (http://www.zeiss.com/us/about/history.shtml)
- The Abbe Principle (*Abbe errors*) resulted from observations about measurement errors in the manufacture of microscopes:
  - *If errors in parallax are to be avoided, the measuring system must be placed coaxially with the axis along which the displacement is to be measured on the workpiece*
    - Strictly speaking, the term *Abbe error* only applies to measurement errors
- When an angular error is amplified by a distance, e.g., to create an error in a machine's position, the strict definition of the error is a *sine* or *cosine* error

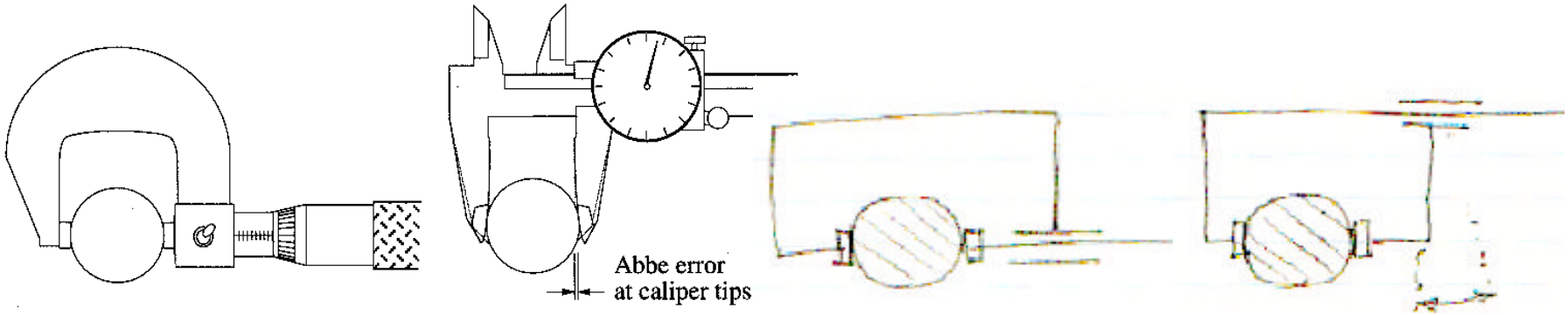


From www.zeiss.com

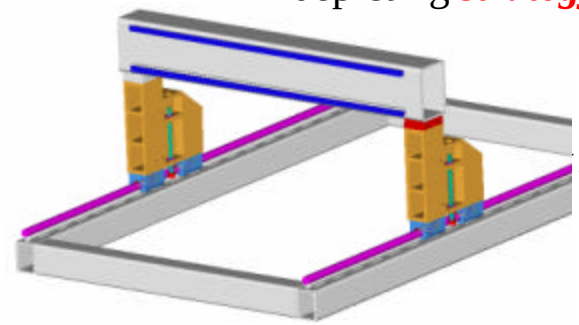


Abbe's Principle: *Locating Components*

- Geometric: Angular errors are amplified by the distance from the source
  - Measure near the source, and move the bearings and actuator near the work!
- Thermal: Temperatures are harder to measure further from the source
  - Measure near the source!



- Thinking of Abbe errors, and the system FRs is a powerful catalyst to help develop DPs, where location of motion axes is depicted schematically
  - Example: Stick figures with arrows indicating motions are a powerful simple means of depicting ***strategy*** or ***concepts***



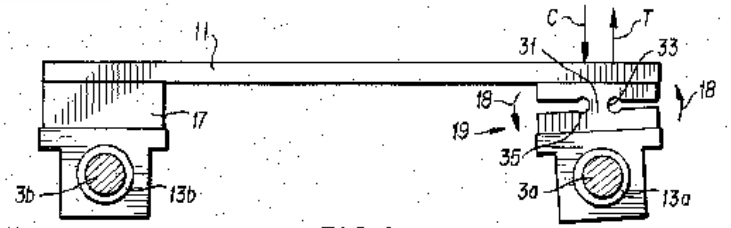
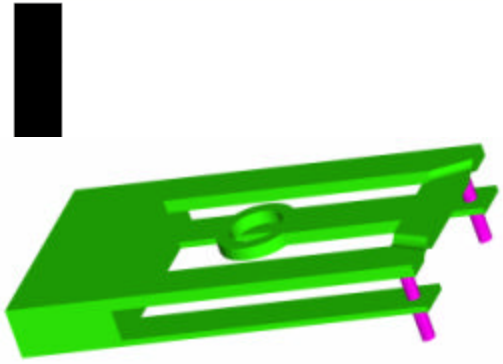
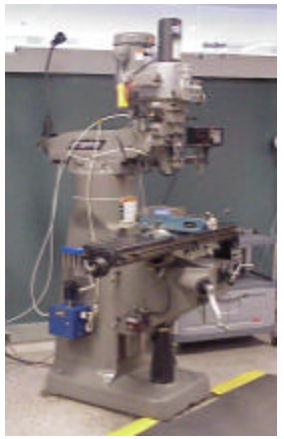


FIG. 3



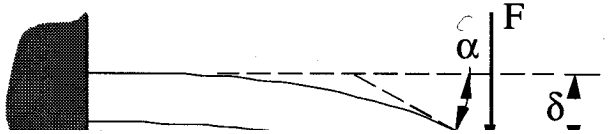
Abbe's Principle: *Cascading Errors*



- A small angular deflection in one part of a machine quickly grows as subsequent layers of machine are stacked upon it…
  - A component that tips on top of a component that tips…
  - If *You Give a Mouse a Cookie*… (great kid's book for adults!)
- Error budgeting keeps tracks of errors in cascaded components
  - Designs must consider not only linear deflections, but angular deflections and their resulting *sine errors*…









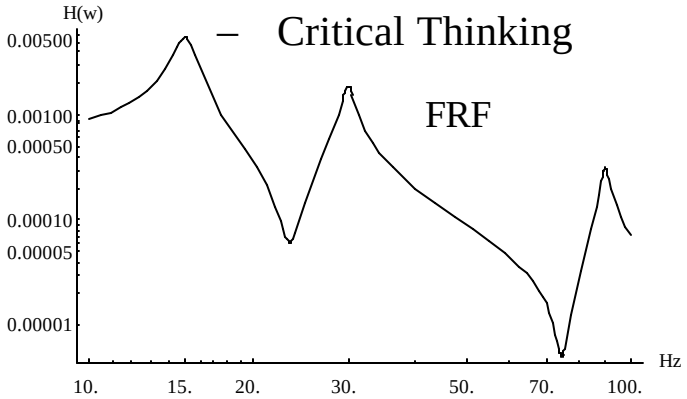


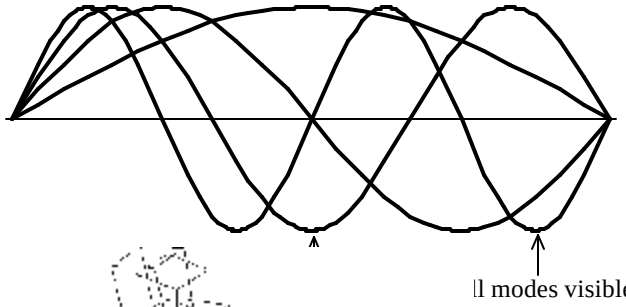


Maxwell & Reciprocity



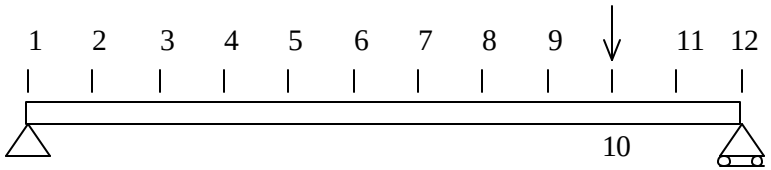
- Maxwell's theory of *Reciprocity*
  - Let *A* and *B* be any two points of an elastic system. Let the displacement of *B* in any direction *U* due to a force *P* acting in any direction *V* at *A* be *u*; and the displacement of *A* in the direction *V* due to a force *Q* acting in the direction *U* at *B* be *v*. Then $Pv = Qu$ (from Roark and Young Formulas for Stress and Strain)
- The principle of *reciprocity* can be extended in philosophical terms to have a profound effect on measurement and development of concepts
  - Reversal
  - Critical Thinking

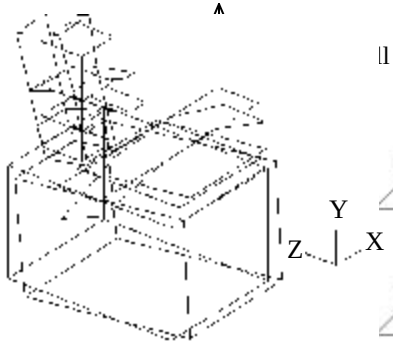


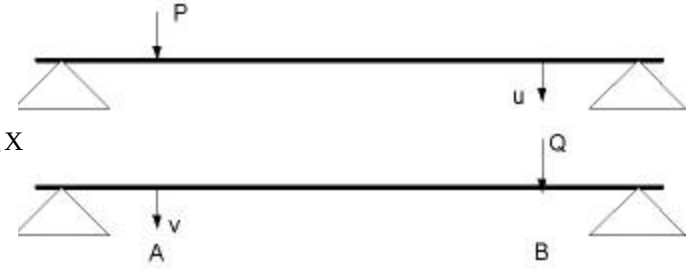




James Clerk Maxwell 1831-1879

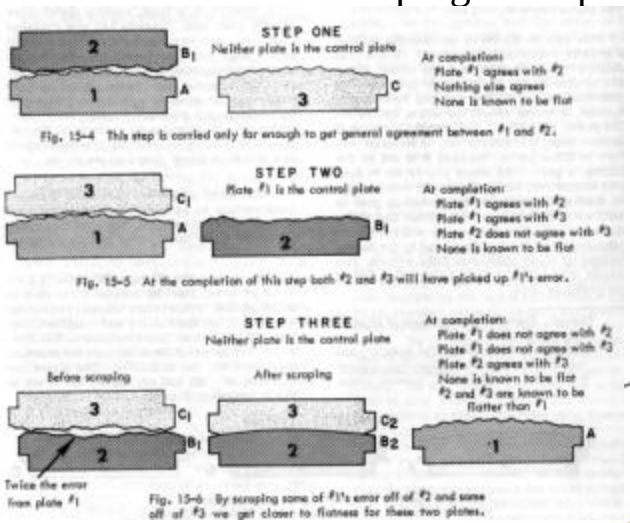






Maxwell & Reciprocity: *Reversal*

- *Reversal* is a method used to take out repeatable measuring instrument errors
  - One of the principal methods by which advances in accuracy of mechanical components have been continually made
- There are many application variations for measurement and manufacturing
  - Two bearings rails ground side-by-side can be installed end-to-end
    - A carriage whose bearings are spaced one rail segment apart will not pitch or roll
  - Scraping three plates flat

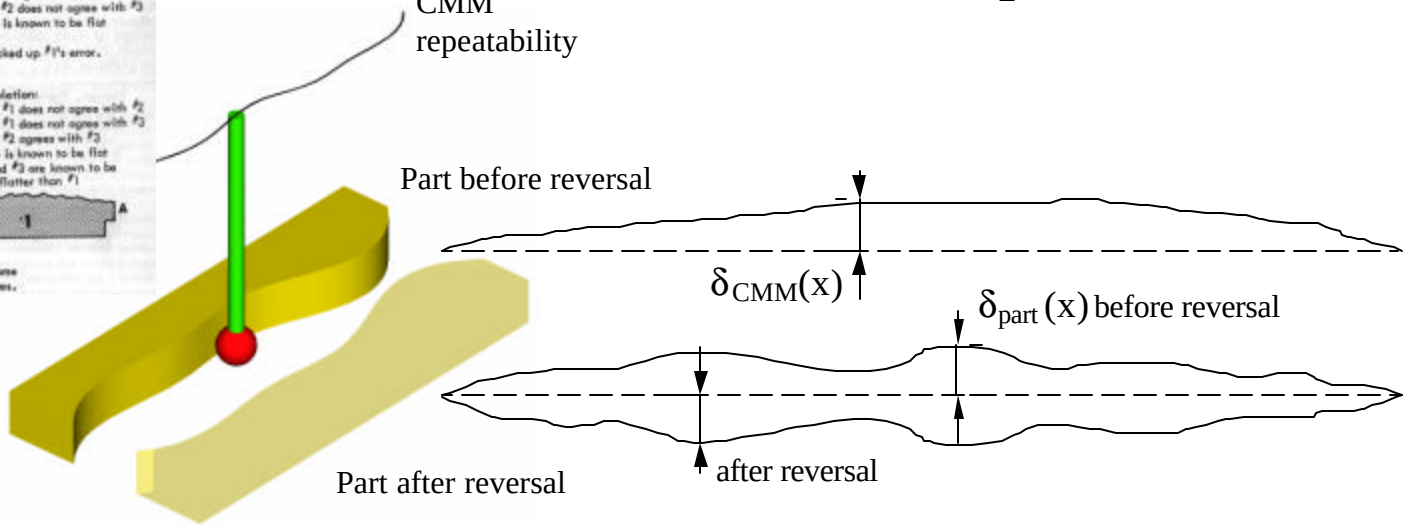


From T. Busch, Fundamentals of Dimensional Metrology, Delmar Publishers, Albany, NY, 1964

$$Z_{\text{probe before reversal}}(x) = \delta_{CMM}(x) - \delta_{part}(x)$$

$$Z_{\text{probe after reversal}}(x) = \delta_{CMM}(x) + \delta_{part}(x)$$

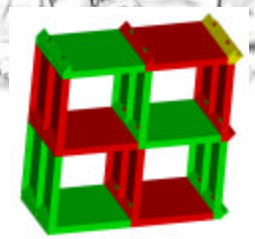
$$\delta_{part}(x) = \frac{-Z_{\text{probe before reversal}}(x) + Z_{\text{probe after reversal}}(x)}{2}$$

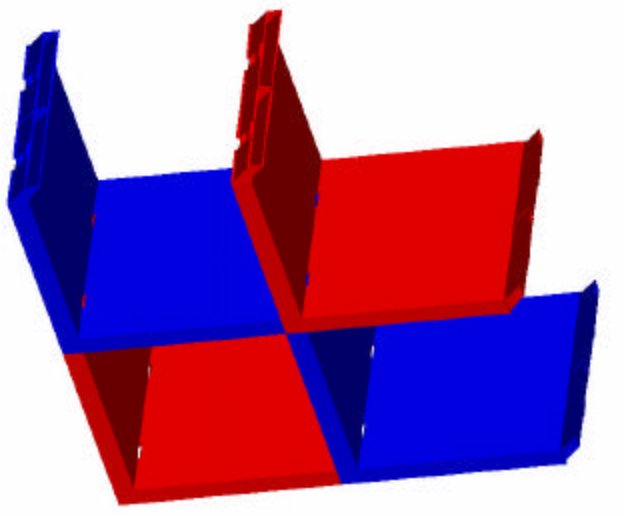


Maxwell & Reciprocity: *Critical Thinking*

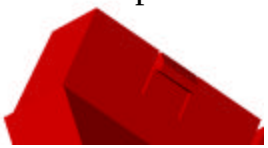
- If you are happy with something, turn it around!
- If you are unhappy with something, turn it around!
- If you are comfortable on your back, still turn over and try lyi ng down on your front……
- You can make a system *insensitive* to its surroundings, or you can *isolate* a system from its surroundings…
- If you cannot solve a problem by starting at the beginning, start at the end and work backwards!
- Example: Snap fit that required a side pull in a mold was literally inversed to eliminate the need for a side pull!
  - CubbeeZ™ modular storage elements
  - US Patent 5,888,114

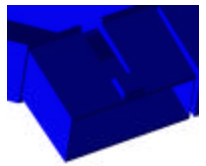






Side pull => No side pull





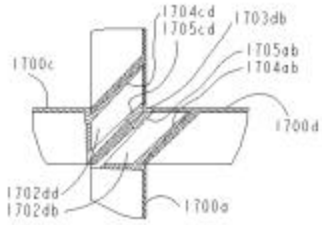


Fig. 9

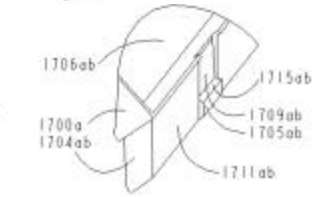


Fig. 10

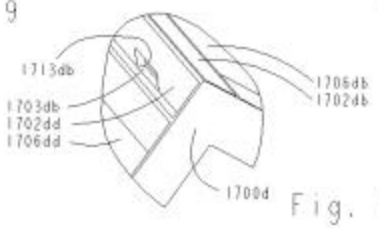
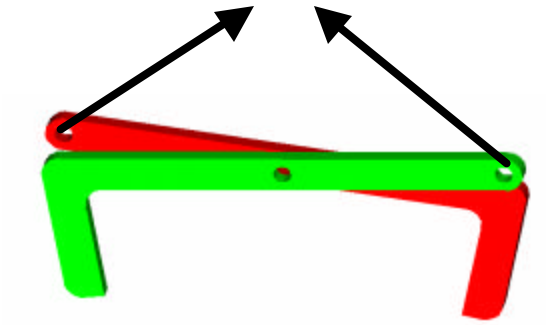


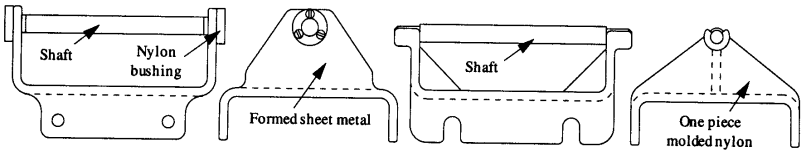
Fig. 11

Self-Principles

- The manner in which a design reacts to inputs determines its output
  - How a design affects itself, can be used to its advantage
  - Reciprocity would philosophically tell us to look for a solution where a potentially detrimental result can be used to cancel the effect
  - Martial artists practice this principle all the time!
- *Self-Help:* A design that uses the inputs to assist in achieving the desired output
  - An initial effect is used to make the device ready for inputs
    - The supplementary effect is that which is induced by the inputs, and it enhances the output
  - *Example:* Airplane doors act like tapered plugs
    - When the door is shut, latches squeeze the seal, making the cabin airtight
    - As the plane ascends and the outside air pressure decreases, the higher inner air pressure presses on the door and causes the door to seal even tighter
  - Example: *Back-to-back* angular contact bearings are thermally stable
- Other *self-principles* similarly exist:
  - *Self Balancing, Self-Reinforcing, Self-Protecting, Self-Limiting, Self-Damaging, Self-Braking, Self-Starting….*



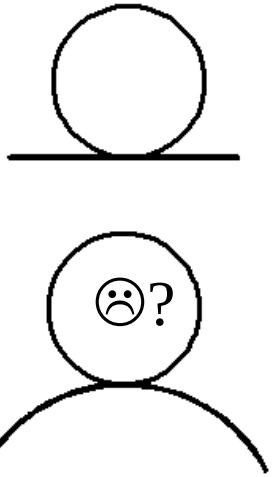
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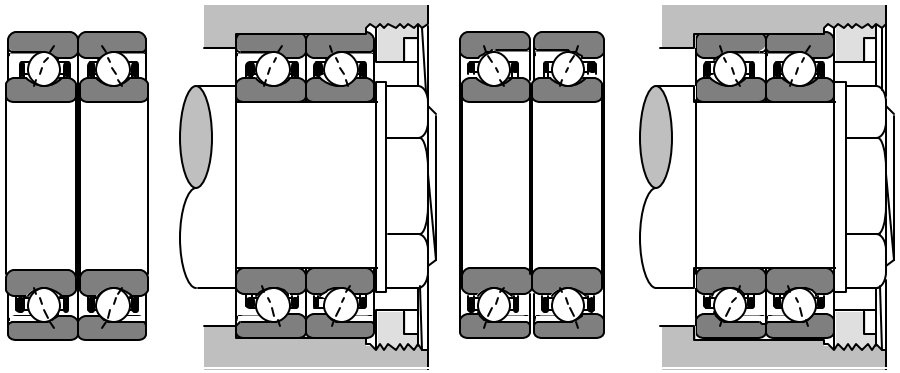


Stability



- All systems are either *stable, neutral, or unstable*
  - Saint-Venant's principle was applied to bearing design to reduce the chance of sliding instability (e.g., a drawer jamming)
  - A snap-fit uses an applied force to move from a stable, to a neutrally stable, to an unstable to a final new stable position
  - Wheels allow a system to roll along a flat surface
  - As the load on a tall column increases, infinitesimal lateral deflections are acted on by the axial force to become bending moments, which increase the deflections….
    - Reciprocity says this detrimental effect can be useful: fire sprinklers are activated by a column that buckles when it becomes soft…
  - *Back-to-back* mounted bearings are intolerant of misalignment, but use axial thermal growth to cancel radial thermal growth for constant preload and thermal stability at high speeds
  - *Face-to-face* mounted bearings are tolerant of misalignment, but axial thermal growth adds to radial thermal growth and causes the bearings to become overloaded and seize at high speeds





Before mounting | Back-to-back mounting after inner rings are clamped together | Before mounting | Face-to-face mounting after outer rings are clamped together

$$\omega_n = k^2\sqrt{\frac{EI}{A\rho L^4}} \qquad F_{buckle} = \frac{cEI}{L^2}$$



	Cantilevered		Simply supported		Fixed-simply supported		Fixed-fixed	
n	k	c	k	c	k	c	k	c
1	1.875	2.47	3.142	9.87	3.927	20.2	4.730	39.5
2	4.694		6.283		7.069		7.853	
3	7.855		9.425		10.210		10.996	
4	10.996		12.566		13.352		14.137	
n	(2n-1)π/2		nπ		(4n+1)π/4		(2n+1)π/2	



Symmetry

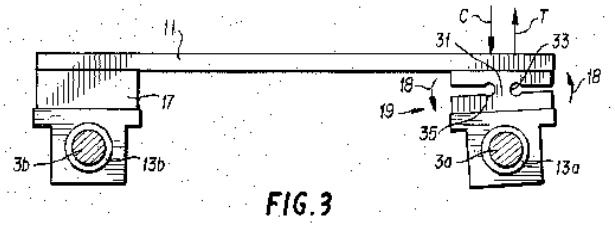
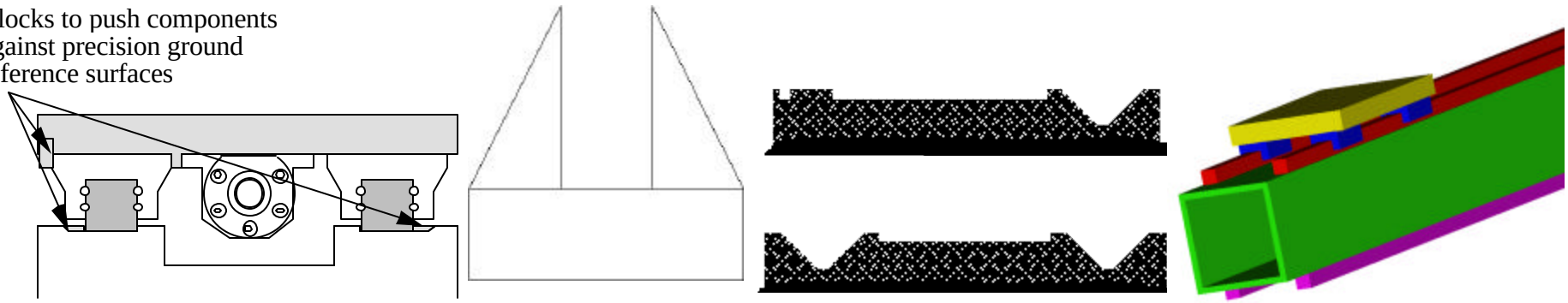


FIG. 3

- *Symmetry* can be a powerful design tool to minimize errors
  - Thermal gradient errors caused by bi-material structures can minimize warping errors
    - Steel rails can be attached to an aluminum structure on the plane of the neutral axis
    - Steel rails on an aluminum structure can be balanced by steel bolted to the opposite side
  - Angular error motions can be reduced by symmetric support of elements
- *Symmetry* can be detrimental (Maxwell applied to symmetry)
  - Differential temperature minimized by adding a heat source can cause the entire structure to heat up
    - Only attempt with extreme care
    - Better to isolate the heat source, temperature control it, use thermal breaks, and insulate the structure
  - A long shaft axially restrained by bearings at both ends can buckle
  - Remember-when you generalize, you are often wrong
    - The question to ask, therefore, is "Can symmetry help or hurt this design?"

Blocks to push components against precision ground reference surfaces



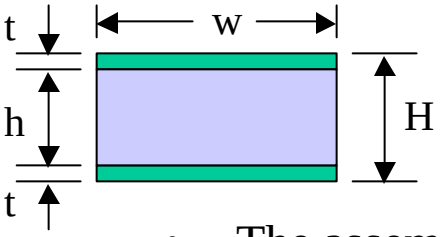
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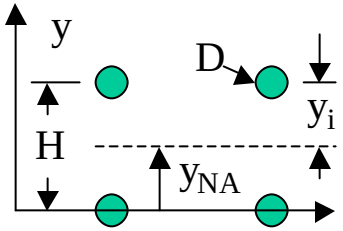
Parallel Axis Theorem



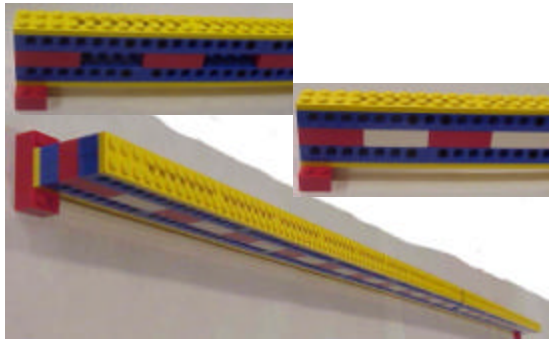
- The *Parallel Axis Theorem* is useful for calculating the moments of inertia for complex objects
  - The stiffness of a design is proportional to the square of the distance y_i of the component structural members' neutral axes from the assembly's neutral axis



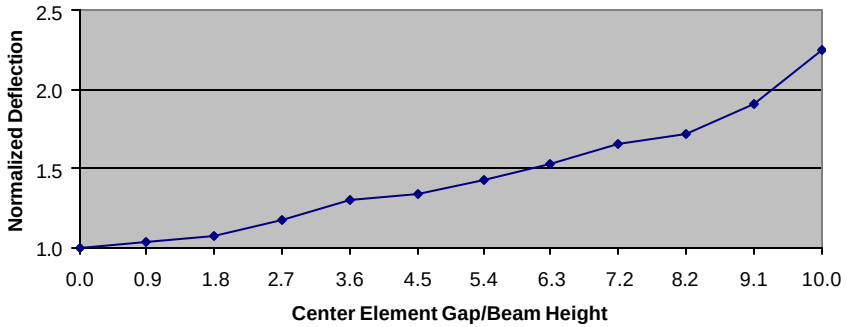
$$I = \sum_{i=1}^{N} I_i + \sum_{i=1}^{N} y_i^2 A_i \qquad y_{NA} = \frac{\sum_{i=1}^{N} y_i A_i}{\sum_{i=1}^{N} A_i}$$



- The assembly's neutral axis is found in the same manner as the center of gravity, and it is located a distance y_{NA} from an arbitrary plane



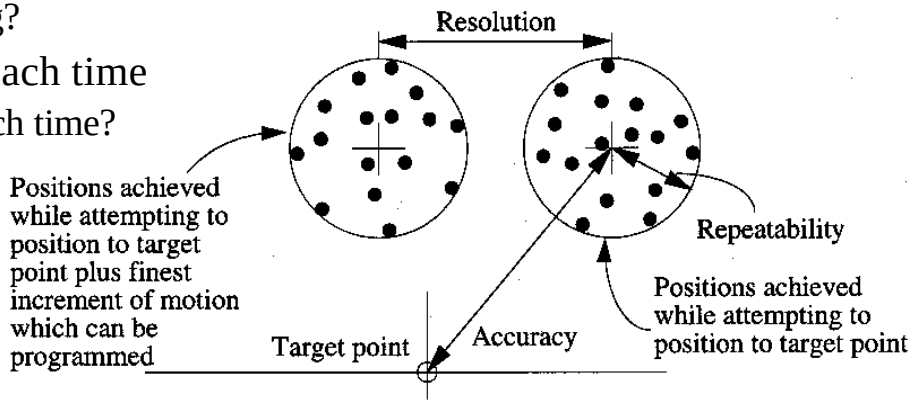




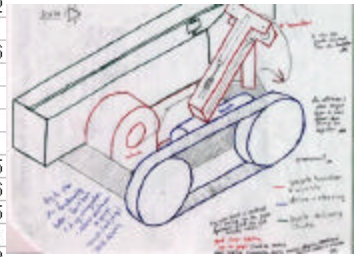
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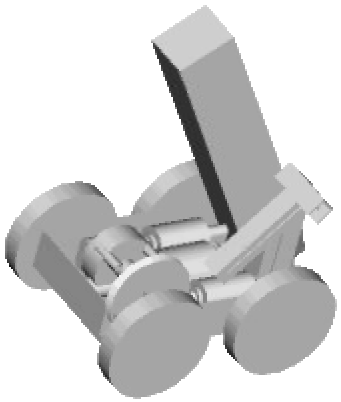
Accuracy, Repeatability, & Resolution

- Anything you design and manufacture is made from parts
  - Parts must have the desired accuracy, and their manufacture has to be repeatable
- *Accuracy:* the ability to tell the truth
  - Can two machines make exactly the same part?
  - Are the parts the exact size shown on the drawing?
- *Repeatability:* the ability to tell the same story each time
  - Can the machine make the exact same motion each time?
  - Are the parts all the same size?
- *Resolution:* the detail to which you tell a story
  - How fine can you adjust a machine?
  - How small a feature can you make?
- How do these affect the design process?

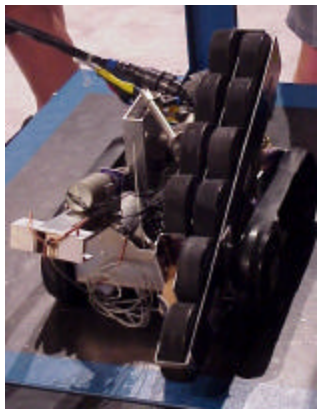


Hook launcher Model	
weight of hook (Kg)	0.05
muzzle velocity	9.4
Number of springs	2
d (draw)	0.095216
Winch model	
radius	0.05
mass	6
w (rpm)	55
torque	2.1
velocity	0.287833





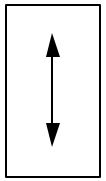




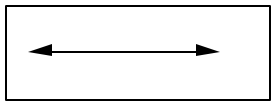


Accuracy, Repeatability, & Resolution: *Mapping*

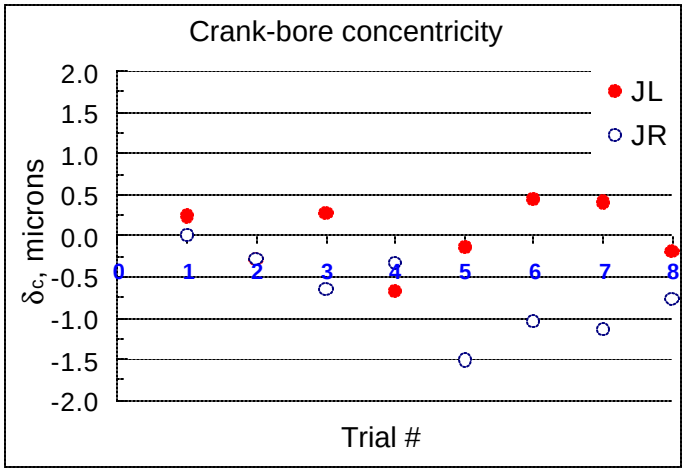
- It is often most important to obtain mechanical *repeatability*, because *accuracy* can often be obtained by the sensor and control system
  - When the error motions of a machine are *mapped*, the controller multiplies the part height by the axis' pitch & roll to yield the sine error for which orthogonal axes must compensate

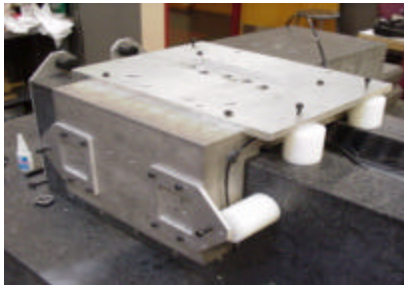


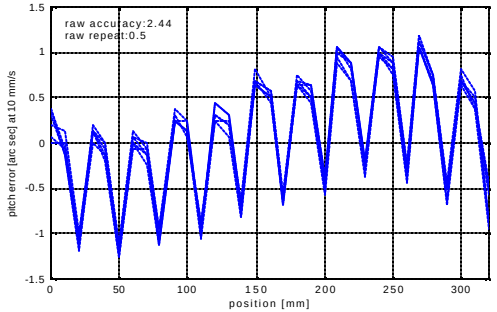
Y axis: Can be used to compensate for straightness errors in the X axis.

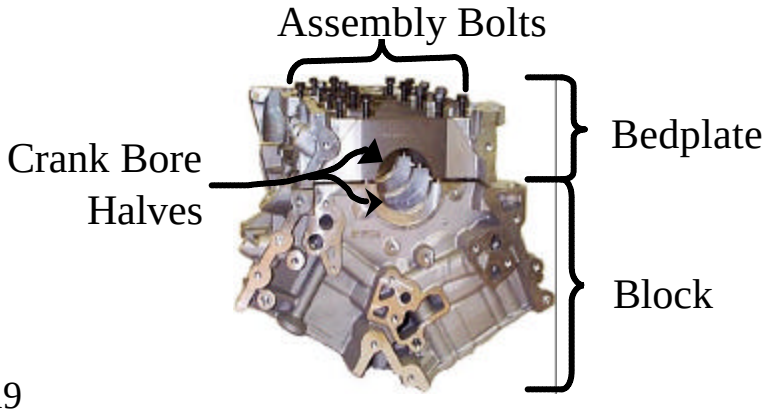


X axis: Can be used to compensate for straightness errors in the Y axis.





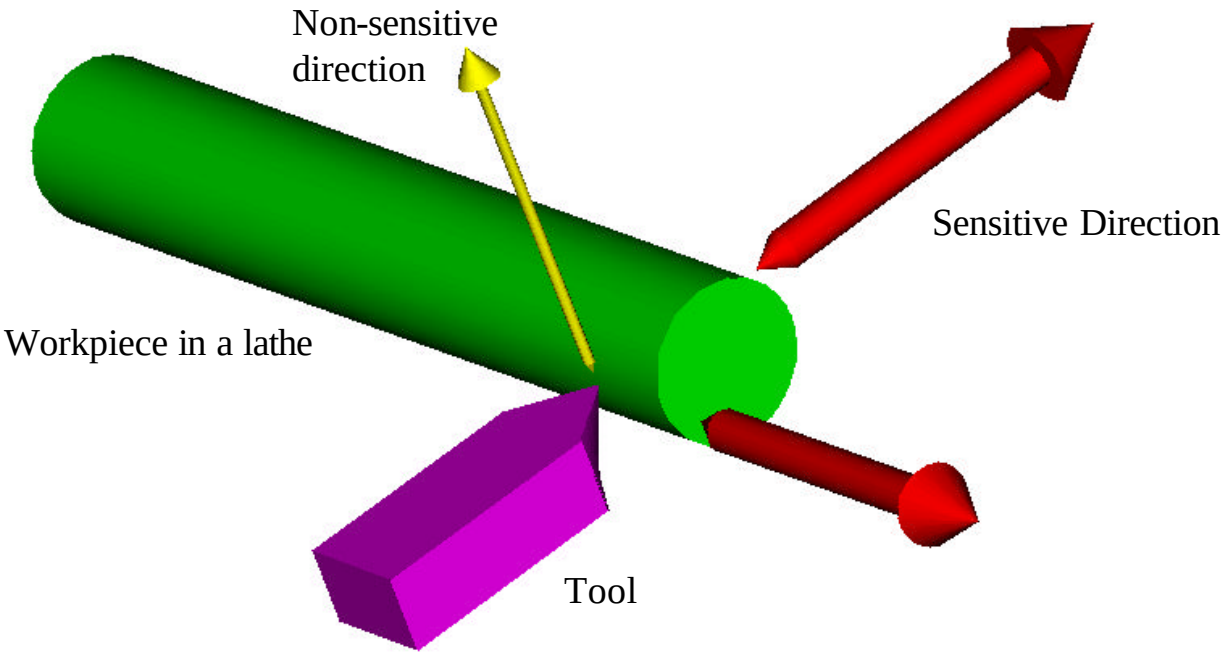


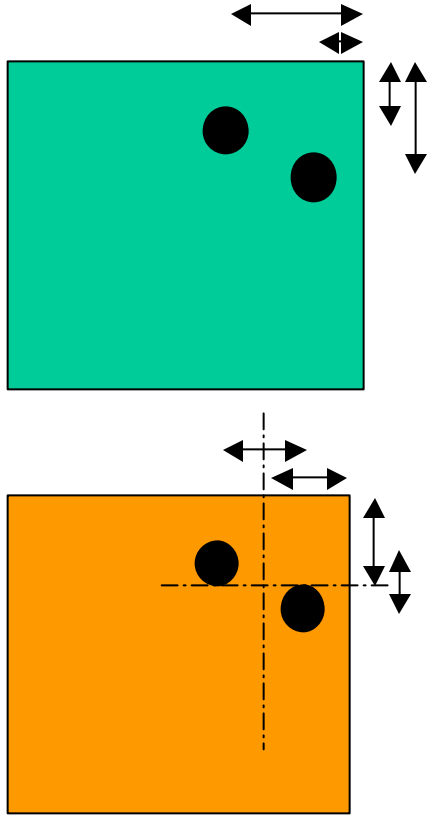


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Sensitive Directions & Reference Features

- In addition to *accuracy*, *repeatability*, and *resolution*, we have to ask ourselves, “when is an error really important anyway?”
  - Put a lot of effort into accuracy for the directions in which you need it
    - The *Sensitive Directions*
    - Always be careful to think about where you need precision!



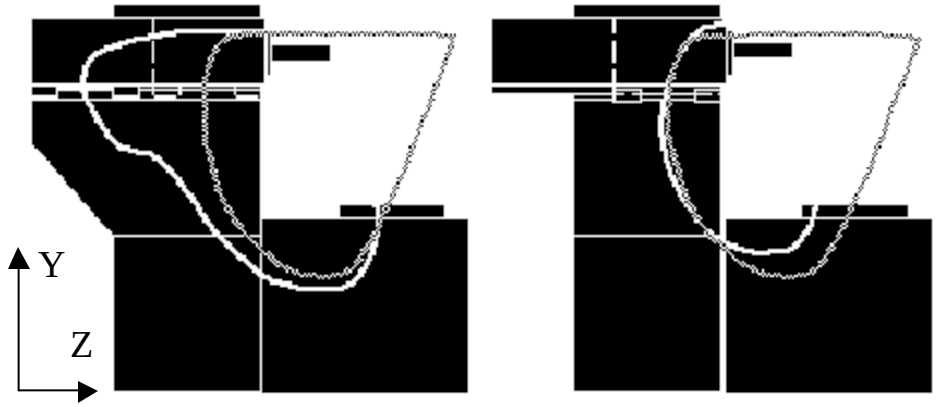


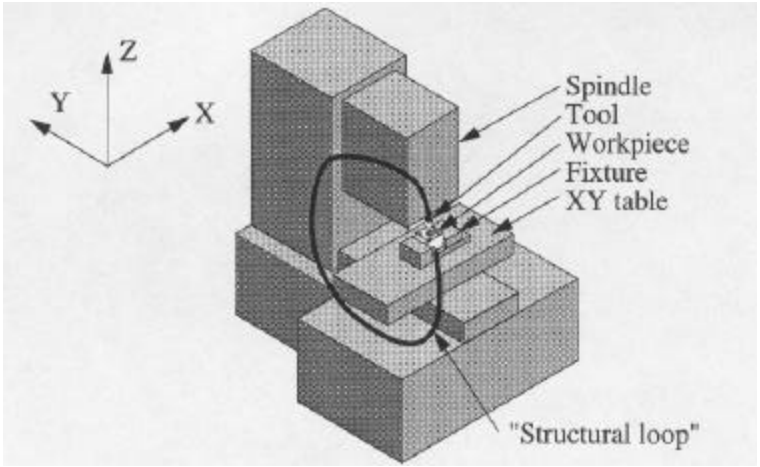
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Structural Loops



- The *Structural Loop* is the path that a load takes as it travels from the tool through the structure and to the work
  - It contains joints and structural elements that locate the tool with respect to the workpiece
  - It can be represented as a stick-figure to enable a design engineer to create a ***concept***
  - Subtle differences can have a HUGE effect on the performance of a machine
  - The *structural loop* gives an indication of machine stiffness and accuracy
    - *The product of the length of the structural loop and the characteristic manufacturing and component accuracy (e.g., parts per million) is indicative of machine accuracy (ppm)*
    - Long-open *structural loops* have less stiffness and less accuracy

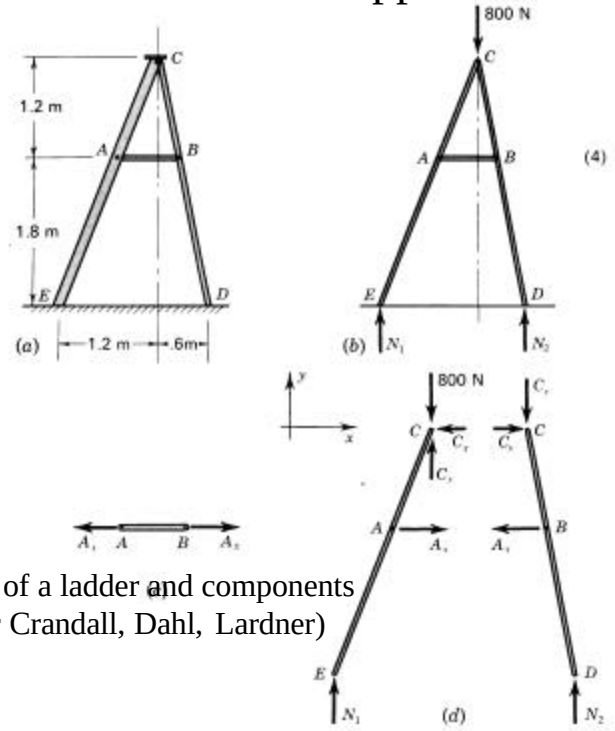


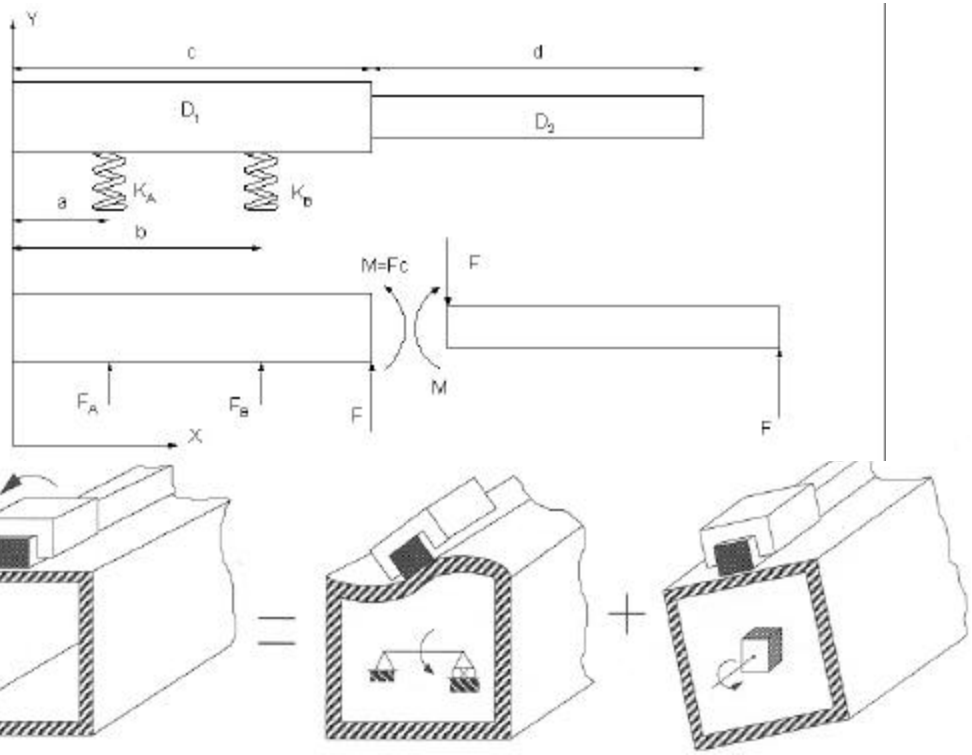


Free Body Diagrams & Superposition



- *Free body diagrams* allow a designer to show ***components*** and their relationship to each other with respect to forces transmitted between them
  - Invaluable for properly visualizing loads on ***components***
  - In order to properly constrain a ***component***, one has to understand how it is loaded and constrained
- *Superposition* allows a complex load to be broken up into ***components*** each of which can be applied one at a time, and then their net effects added

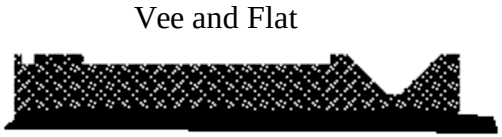


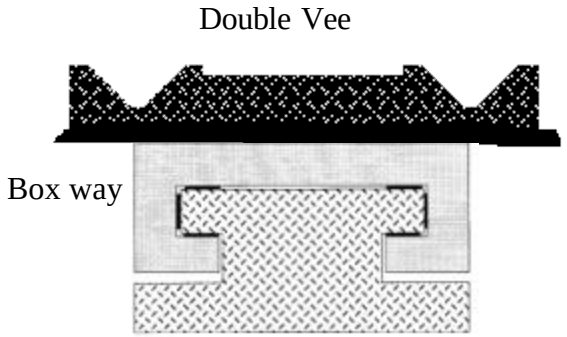


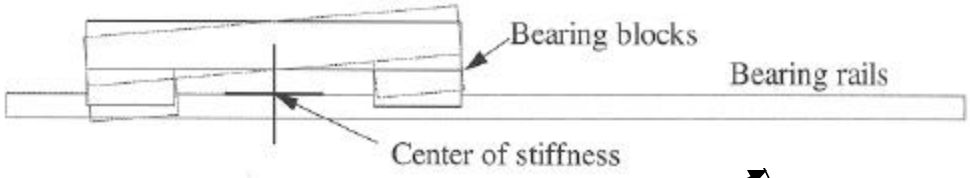
FBD of a ladder and components (after Crandall, Dahl, Lardner)

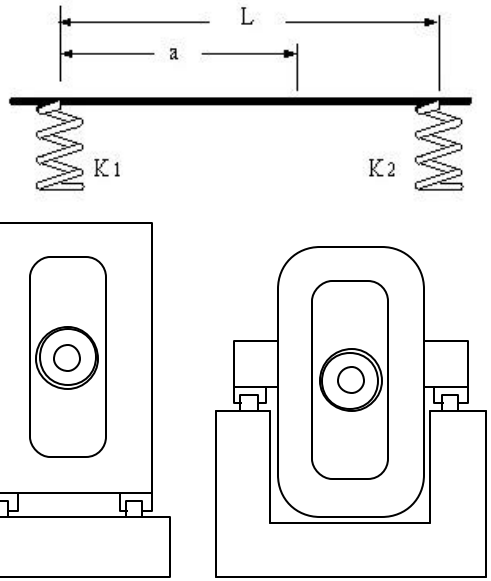
Centers-of-Action

- The *Centers-of-Action* are points at which when a force is applied, no moments are created:
  - *Center-of-Mass*
  - *Center-of-Stiffness*
  - *Center-of-Friction*
  - *Center-of-Thermal Expansion*
- A system is most robust when forces are applied as near as possible to the *Centers-of-Action*

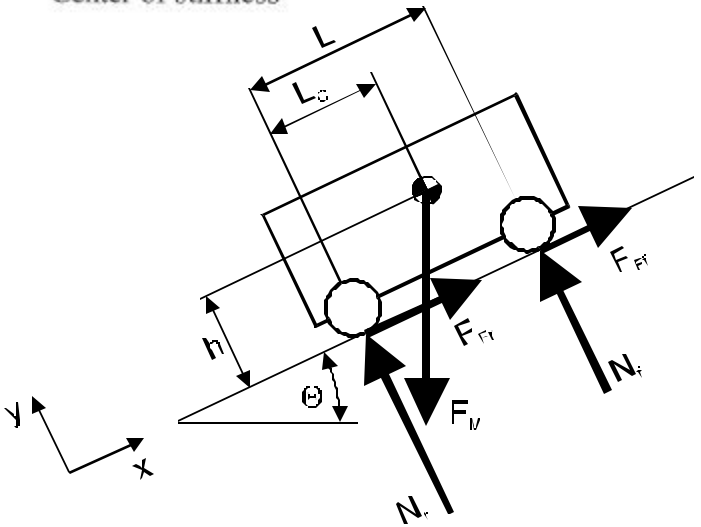






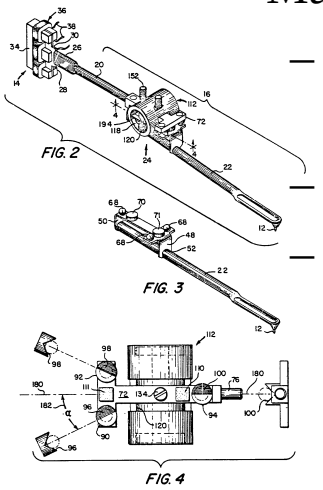




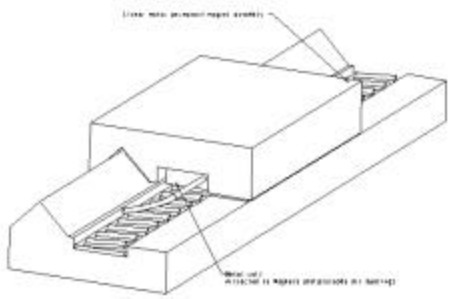


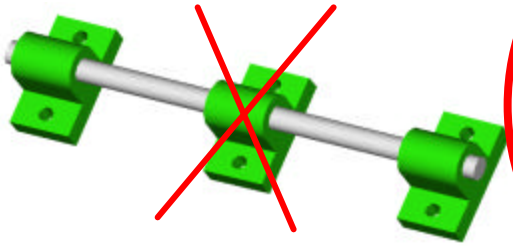
Exact Constraint Design

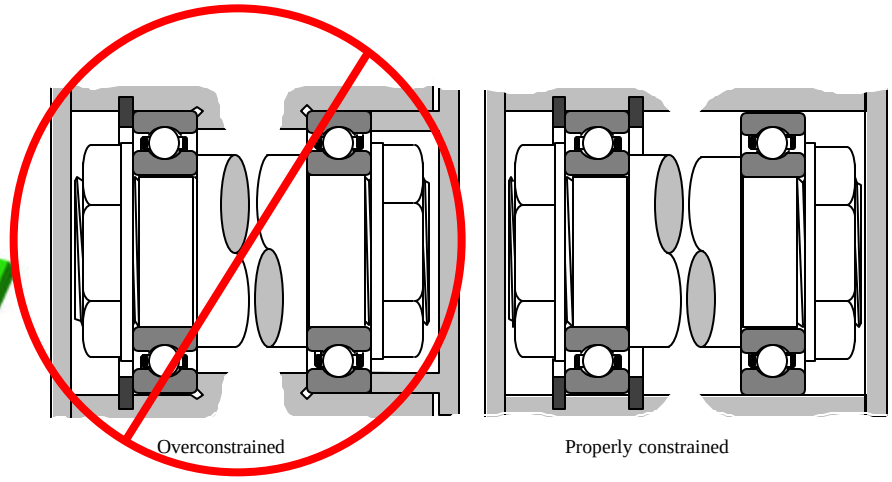
- Every rigid body has 6 *Degrees of Freedom* (DOF)
  - An exactly constrained design has no chance of deforming or having its function impaired be it by assembly, fastener tightening, thermal expansion, or external loads
- Make sure you have constrained what you want to constrain!
  - For a body to have N degrees of freedom free to move, there must be 6-N bearing reaction points!
  - To resist translation, a force is required.
  - To resist rotation, a moment, or two forces acting as a couple, is required!
    - Saint-Venant rules! Do not constrain a shaft with more than 2 bearings, unless it is very long…







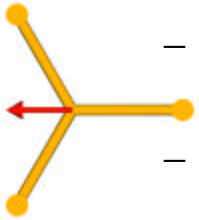


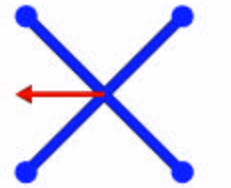


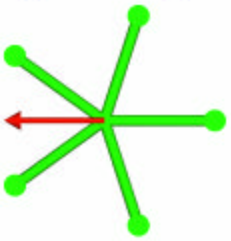
Elastically Averaged Design

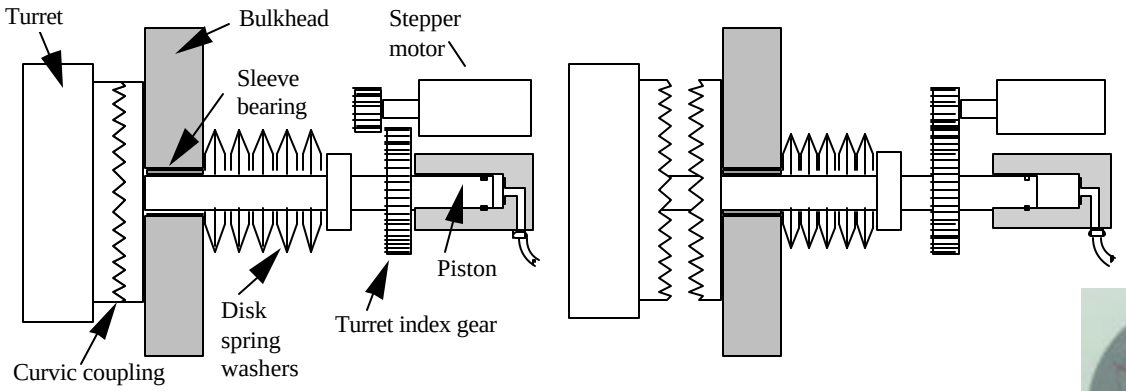


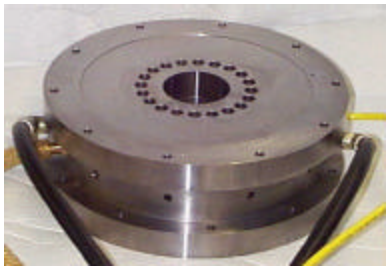
- Applying *Reciprocity* to *Exact Constraint Design* implies that instead of having an exact number of constraints, have an "infinite" number of constraints, so the error in any one will be averaged out!
  - Indexing tables often use a *curvic* coupling (two face gears meshed together) to achieve a high degree of elastic averaging (accuracy), stiffness, and load capacity
  - Windshield wiper blades and surface plate or large-mirror supports distribute the loads using *wiffle trees*

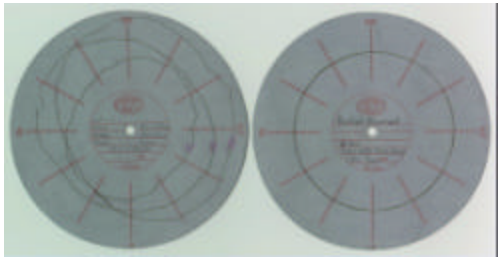


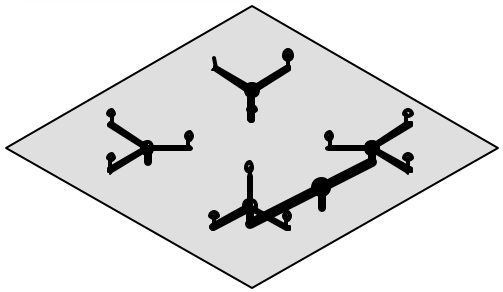




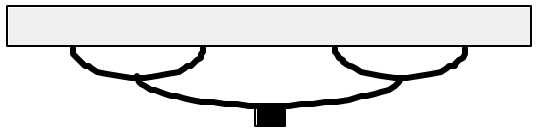






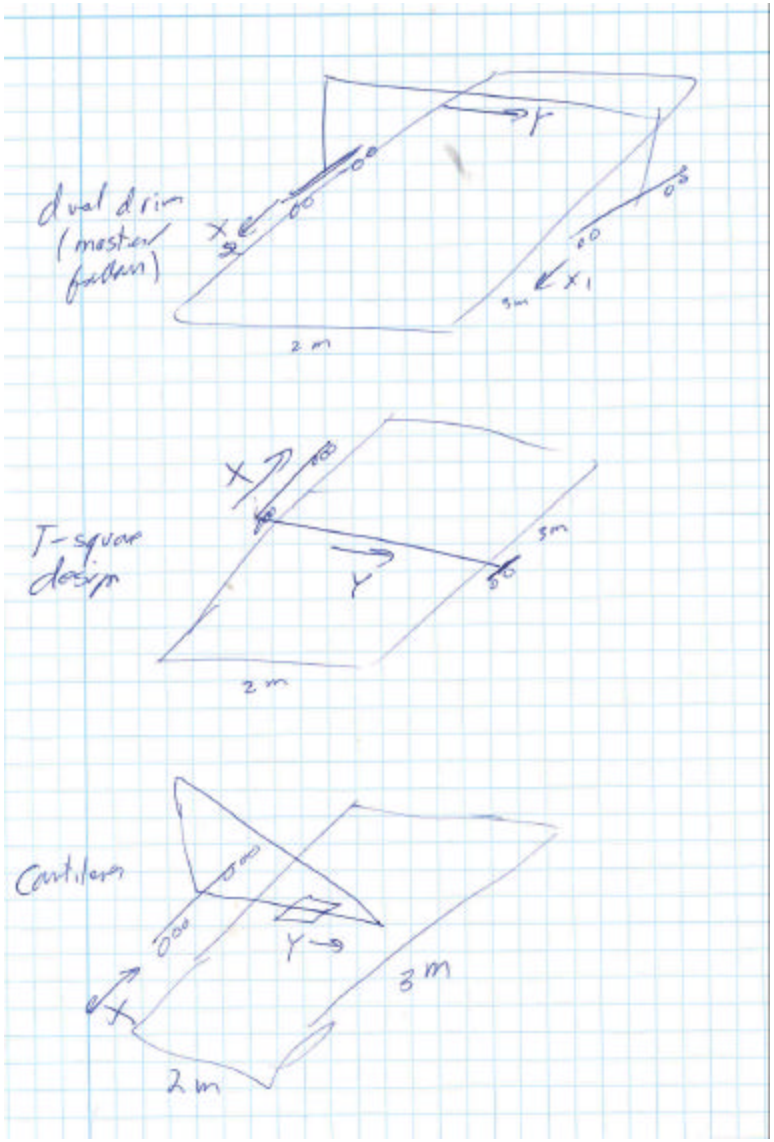






Stick Figures

- Use of *fundamental principles* allows a designer to sketch a machine and error motions and coordinate systems just in terms of a *stick figure:*
  - The sticks join at centers of stiffness, mass, friction, and help to:
    - Define the sensitive directions in a machine
    - Locate coordinate systems
    - Set the stage for error budgeting
  - The designer is no longer encumbered by cross section size or bearing size
    - It helps to prevent the designer from locking in too early
- Error budget and preliminary load analysis can then indicate the required stiffness/load capacity required for each “stick” and “joint”
  - Appropriate cross sections and bearings can then be deterministically selected
- It is a “backwards tasking” solution method that is very very powerful!



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